ELEMENTS OF $\mathcal{PT}$-SYMMETRIC QUANTUM MECHANICS AND THEIR MATHEMATICAL INTERPRETATION IN THE KREIN'S SPACES THEORY.

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The use of non-Hermitian operators and indefinite Hilbert space structures in quantum mechanics dates back to the early 1940s. However, the interest in this subject strongly increased after it had been discovered numerically by C.M. Bender and S. Boettcher in 1998 that complex Hamiltonians

$$A_\varepsilon = -\frac{d^2}{dx^2} + x^2(ix)^\varepsilon, \qquad \varepsilon \geq 0$$

have real spectrum (like a self-adjoint operator). The reality of the spectrum of A_ε is a consequence of its $\mathcal{PT}$-symmetry, which means that $\mathcal{PT}A_\varepsilon = A_\varepsilon\mathcal{PT}$, where $\mathcal{P}$ is the space reflection operator (parity) and $\mathcal{T}$ is the (antilinear) complex conjugation. This gave rise to a consistent complex extension of conventional quantum mechanics into $\mathcal{PT}$ quantum mechanics (PTQM), and ensure a steady interests to mathematically correct and rigorous analysis of non-self-adjoint operators arising in $\mathcal{PT}$-symmetric setting.

The aim of the report is to present some underlying structures of PTQM and to discuss the possibility of their mathematical treatment within the Krein's spaces framework (linear spaces with indefinite metric).