# APPLICATIONS OF SPECTRAL METHODS IN CONTROL THEORY

PIOTR GRABOWSKI

ABSTRACT. We examine the closed–loop state operator of a system consisting of the $\mathfrak{RC}$ electric transmission line controlled, in the positive feedback, by the proportional controller with gain coefficient $K \in \mathbb{R}$,

$$\mathcal{A}_K x = x'', \quad D(\mathcal{A}_K) = \{x \in : \ x'(1) = 0, x(0) = Kx(1)\} \ .$$

Using the direct spectral approach based on a discrete version of the spectral theorem and the Riesz basis concept we establish that it generates an **EXS** analytic semigroup on $\mathrm{H} = \mathrm{L}^2(0,1)$, provided that $-\cosh\pi < K < 1$, $K \neq -1$.

Next we compare this result with the indirect spectral approach based on a new result, in spirit of the Weiss-Staffans perturbation theorem, which relies on treating the closed-loop operator

$$\mathcal{A}^c x = \mathcal{A}_0 \left[x - kdc^\# x\right], \quad D(\mathcal{A}^c) = \left\{x \in D(c^\#) : \ x - kdc^\# x \in D(\mathcal{A}_0)\right\}$$

as a feedback perturbation of the open–loop state operator $\mathcal{A}_0$, $k = -K$.

For that we assume that (**i**) $\mathcal{A}_0$ generates an **EXS** analytic semigroup, (**ii**) the vectors $d$, $h$, where $h^* = c^\# \mathcal{A}_0^{-1}$, give rise to the so–called *conjugate Balakrishnan-Washburn estimates*, (**iii**) the transfer function $\hat{g}(s) := sc^\#(sI - \mathcal{A}_0)^{-1}d - c^\# d$ is in $\mathrm{H}^\infty(\mathbb{C}^+) \cap \mathrm{H}^2(\mathbb{C}^+)$, (**iv**) $-\frac{1}{k} \notin \overline{\hat{g}(\mathbb{C}^+)} \cap \mathbb{R}$ and (**v**) $c^\# d = d^\# h$, where $d^\#$ is an extension of $d^* \mathcal{A}_0^*$ from $D(\mathcal{A}_0^*)$ onto $D(d^\#)$ such that $h \in D(d^\#)$.

This result applied to controlled $\mathfrak{RC}$ electric transmission line with negative proportional feedback of gain $k$ allows to conclude that $\mathcal{A}^c$ generates an **EXS** $\mathrm{C}_0$–semigroup for $k \in (-1, \cosh\pi)$ though an open question is how to prove the analyticity of this semigroup.

Notice that the direct spectral approach suggests an affirmative answer to this question.

INSTITUTE OF CONTROL AND BIOMEDICAL ENGINEERING, AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY, A. MICKIEWICZ AVENUE 30/B1, RM.314, PL 30-059 CRACOW, POLAND

*E-mail address*: pgrab@agh.edu.pl

*Date*: To be presented on November 19, 2014 at Seminar of the Chair of Mathematical Analysis, Numerical Mathematics and Probabilistic Methods, Faculty of Applied Mathematics, AGH UST.

*Key words and phrases.* control of infinite–dimensional systems, semigroups, Weiss–Staffans perturbation theorem.