

Krein space methods for indefinite Sturm-Liouville operators

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We consider singular Sturm-Liouville operators with an indefinite weight, i.e. operators of the form

$$A = \operatorname{sgn}(\cdot) \left(-\frac{d^2}{dx^2} + V \right) \quad (1)$$

on \mathbb{R} . It is assumed that V is a real-valued, locally integrable potential such that the limits $\lim_{x \rightarrow \pm\infty} V(x)$ exist and are finite.

Closely related to the operator A in (1) is the *definite* Sturm-Liouville operator

$$B = -\frac{d^2}{dx^2} + V,$$

which is selfadjoint and semi-bounded in the Hilbert space $L^2(\mathbb{R})$.

Our aim is to describe the spectrum of the indefinite Sturm-Liouville operator A . We will discuss different cases which are organized according to the location of the spectrum $\sigma(B)$ of the definite Sturm-Liouville operator B and the essential spectrum $\sigma_{ess}(B)$ of B .