In this talk we discuss several examples of Schrödinger operators which can exhibit unexpected spectral properties, in particular, they may have a discrete spectrum despite the fact that the corresponding phase space volume is infinite. The first example concerns the operator with the potential $|xy|^p-\lambda(x^2+y^2)^{p/(p+2)}$, which is a perturbation of the well-know model $-\Delta+|xy|^p$. We show that the spectrum may be purely discrete even for potentials unbounded from below, however, it exhibits a so called "spectral transition" as the coupling constant λ changes. As a second example we consider a regular version of the so-called Smilansky model which demonstrates the similar spectral behavior. In the last example, we study Schrödinger operators in cusp-shaped regions and in tubes and derive inequalities of Berezin-Lieb-Yau type in terms of the geometry of the regions.