Hankel operators and rational approximation of singular functions

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We show that all Hankel operators H realized as integral operators with kernels $h(t+s)$ in $L^2(\mathbf{R}_+)$ can be quasi-diagonalized as $H = L^*\Sigma L$. Here L is the Laplace transform, Σ is the operator of multiplication by a function (distribution) $\sigma(\lambda)$, $\lambda \in \mathbf{R}$, linked to h by the formula $h = L^*\sigma$. The sigma-function of a Hankel operator contains substantial information about its spectral properties and, in particular, allows one to find the asymptotic behaviour of its singular values.

As an application of this construction, we consider rational approximation of functions ω with logarithmic singularities and find an asymptotic formula for the distance in the BMO norm between ω and the set of rational functions of degree n as $n \to \infty$.