Krein space related physics

Uwe Günther
Helmholtz-Zentrum Dresden-Rossendorf

Abstract:
Physical models with anti-linear symmetries can often be described by differential operators self-adjoint in suitably chosen Krein spaces. We briefly comment on the spectral properties of some specific operators self-adjoint in Krein spaces and related effects:
- the operator of the hydrodynamic Squire equation, its scaling behavior and mapping to the operator of the Bender-Boettcher model of $\mathcal{PT}$ Quantum Mechanics,
- the cusp-type spectral properties in the vicinity of third-order exceptional points (algebraic branch points),
- the unfolding of higher-order exceptional points of the spectrum of Hamiltonians in $\mathcal{PT}-$symmetric Bose-Hubbard models described with the help of Puiseux series expansions and Newton polygon techniques.
We briefly explain the basic features of the so-called quantum brachistochrone problem for Hamiltonians self-adjoint in Hilbert spaces and in Krein spaces and demonstrate their interrelation geometrically in terms of contraction-dilation maps in projective Hilbert spaces and via positive operator-valued measures (POVMs) and Naimark dilation. Finally, we briefly comment on recent experimental findings in $\mathcal{PT}-$symmetric (i.e. Krein-space related) physics, especially in optical wave-guide systems and microwave cavities.