Well-posedness for damped hyperbolic equation with critical Hartree type nonlinearity

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In this paper, we study the well-posedness of the initial boundary value problem for the hyperbolic equation with strong damping, weak damping and Hartree type nonlinearity with Hardy-Littlewood-Sobolev critical exponent. First, we establish the local existence of the solution by the Galerkin method and Banach fixed point theorem. Overcoming the challenge from the damping structure of the equation, the nonlocal characteristic of the Hartree type nonlinearity and the lack of compactness caused by the critical exponent, we construct the corresponding potential well theory and prove the invariance of some manifolds. In the framework of the potential well theory, we obtain the global existence and finite time blowup of solutions at the subcritical initial energy level $E(0) < d$ and the critical initial energy level $E(0) = d$, and the finite time blowup of the solution at the arbitrary positive initial energy level $E(0) > 0$. Then, we further describe the dynamical behaviors of the global solution and blowup solution, involving the asymptotic behavior (exponential decay) of the global solution and blowup time estimates (lower and upper bounds) of the blowup solution. Finally, we investigate the stability, i.e., continuous dependence of the global solution on the initial data by applying the decay property of energy, and the obtained continuous dependence result reflects the dissipative nature of the model. In the proof of the continuous dependence, the common energy decay estimate is not applicable, hence we establish a new version of effective energy decay estimate to serve the continuous dependence of the solution on the initial data. We also extend this method to the continuous dependence of the solution on the coefficients of strong damping and weak damping, and the corresponding decay estimates are different from that for the continuous dependence of the solution on the initial data.