

Transitions from one- to two-dimensional dynamics

Dyi-Shing Ou

AGH University of Science and Technology

Abstract

Studies have shown that the dimension of a dynamical system may restrict its possible dynamical behavior. Here, we investigate how the constraints relieve as the dimension increases. In particular, we introduce a topological model which applies to the Hénon and the Lozi families, and view the families as perturbations of the unimodal and the tent families in two dimensions respectively. We use the model to explain the following phenomena:

1. A two dimensional system can have infinitely many sinks (Newhouse, 1974), whereas a one dimensional system can not (Singer, 1978).
2. The kneading theory (Milnor and Thurston, 1988) breaks down in the Hénon and the Lozi families.
3. There are no Fibonacci maps (Lyubich and Milnor, 1993) in two dimensions.