When all closed sets are α-limit sets

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Abstract

Limit sets of forward trajectories, known as ω-limit sets, play an important role in topological dynamics. They are well-studied, even a complete topological characterization of them is known for continuous maps in some spaces like interval or topological graph. On the other hand very little is known about α-limit sets of non-invertible maps. This is perhaps related to the fact that α-limit sets are rather flexible objects - other kinds of limit sets, like ω-limit sets and branch α-limit sets are always internally chain transitive, and this leads to strong topological restrictions on what sets can arise this way. The only obvious general facts about the α-limit sets are that they must be closed and forward invariant. It seems that the problem of characterizing α-limit sets, even in simple topological spaces, has not yet been considered. The main goal of this talk is to answer the question: When are all closed subsets of a topological space X also α-limit sets? We show that the answer is positive for every space with enough arcs, by which we mean a non-degenerate metric space X in which each non-degenerate proper closed subset can be joined to its complement by an arc in X. This includes several important classes of connected spaces, such as graphs, (local) dendrites, connected metrizable manifolds, connected metrizable CW-complexes. Also some disconnected spaces including all zero-dimensional separable metric spaces, in particular, the Cantor space, has the property that every closed subset can be realized as an α-limit set.