Abstract

Fix $m \in \mathbb{N}$. Let (X, ϱ) be a complete and separable metric space and $(\Omega, \Sigma, \mathbb{P})$ be a probability space. We consider a *generalized iterated function system with probabilities* (GIFSP), i.e. $\mathbb{P}$–continuous family $\mathcal{F} = \{f_\omega : X^m \to X : \ \omega \in \Omega\}$.

Given an infinite word $\omega^\infty = (\omega_n)_{n \in \mathbb{N}} \in \Omega^\infty$ we define a (random) ω^∞–*trajectory* of a GIFSP $\mathcal{F}$ starting from $x_1, \ldots, x_m \in X$ as a sequence $(x_n)_{n \in \mathbb{N}}$ of points from X defined by

$$x_{n+m} = f_{\omega_n}(x_n, \ldots, x_{n+m-1}) \quad \text{for} \ \ n \in \mathbb{N}. \tag{1}$$

We would like to study the long–time behavior of random trajectories (1) of a GIFSP with arbitrary number of transformations. To do this we discover the theory of limiting behavior for the pretty large class of Markov–type operators acting on the product of spaces of finite Borel measures on X. We show the nice criterion on the existence of the unique invariant measure for such operator (asymptotic stability). Moreover, we prove that this unique measure is a probabilistic one with the first moment finite, it attracts all trajectories of distributions under the action of the operator in the sense of the Hutchinson–Wasserstein norm as well as in the weak topology. The rate of convergence is geometric.