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On a functional–difference inclusion induced by some iterated function system

(joint work with Grzegorz Guzik)

In *Difference equations and their applications*, Part II, Chapter 1 wrote by A. N. Sharkovsky, Y. L. Maistrienko, E. Yu. Romanenko the following functional–difference equations with continuous–time argument

$$x(t+1) = f(x(t)) \tag{1}$$

is considered. In general, here above f is a given continuous function of some compact space X into itself and $x : [0, \infty) \to X$ is unknown. One can see that solutions of (1) depend on an arbitrary function. More precisely, having a function $\varphi : [0, 1] \to X$ and putting

$$x|_{[0,1]} := \varphi \tag{2}$$

one can get uniquely the solution x on the whole semiline $[0, \infty)$ by the formula

$$x(t+n) = f^n \circ \varphi(t) \quad \text{for } n \in \mathbb{N}. \tag{3}$$

In particular, if φ is a continuous function and the initial condition

$$\varphi(1) = f(\varphi(0)). \tag{4}$$

is assumed, the solution x of (1) is continuous.
The most interesting aspect is a study of asymptotic behavior of graphs of a sequence $(f^n \circ \varphi)$ of compositions of iterates f^n with an 'observable' function φ in a proper Hausdorff metric or the topology of uniform convergence. In many cases such graphs tend to the graph of some set–valued function (with closed graph) rather then to the graph of a single–valued continuous function. Moreover, it is remarkable that usually the asymptotic behavior of such a sequence of graphs strongly depends on a choice of a function φ.
In various models of real systems the given function is not uniquely determined but either can be chosen from some family (finite or not) by a deterministic or random algorithm, or it is determined with some error. Hence it seams quite reasonable to consider the following inclusion

$$x(t+1) \in f(x(t)) \tag{5}$$

instead of the equality (1). Here $f : X \rightsquigarrow X$ is a given set–valued function which can represent a union of all functions possibly chosen.
In the present talk we consider the inclusion (5) with set–valued function f generated by an iterated function system, i.e. a finite family of continuous mappings of X into itself. Which is assumed to possess an attractor (a fractal set). Our main result says that under quite typical assumptions on a given iterated function system we get a simple asymptotic behavior of graphs of compositions $f^n \circ \varphi$, $n \in \mathbb{N}$ which is, surprisingly, independent on a choice of an 'observable' φ.