Dendrites and measures with discrete spectrum

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Möbius disjointness conjecture proposed by Sarnak in 2009 ([2]) states that Möbius function is linearly disjoint from any dynamical system with zero dynamical entropy. The conjecture was confirmed on various one-dimensional spaces, such as interval, circle, topological graphs and some dendrites. It is also known that if all invariant measures have discrete spectrum, then the conjecture holds.

In the talk we focus on the following question stated in [1]:

Which one-dimensional continua X have the property that every invariant measure of (X, f) has discrete spectrum, assuming f is a zero-entropy map?

In particular we focus on dendrites with countable closure of the set of endpoints. We show that for such dendrites every recurrent point is minimal, which is the generalization of the property of zero-entropy interval maps. Then we prove that all invariant measures of zero-entropy maps of such dendrites have discrete spectrum, which confirms the Möbius disjointness conjecture on those spaces. Those results almost completely characterize dendrites for which all invariant measures of zero-entropy maps have discrete spectrum, leaving the case of dendrites with endpoint set countable with uncountable closure unsolved.

All results presented during the talk are obtained as a joint work with J. Hantáková, J. Kupka, P. Oprocha and S. Roth.

References

[1] J. Li, P. Oprocha, G. Zhang, *Quasi-graphs, zero entropy and measures with discrete spectrum* Nonlinear. (2022) **35** no. 3, 1360–1379,

[2] P. Sarnak, *Three lectures on the Möbius function randomness and dynamics*, https://www.math.ias.edu/files/wam/2011/PSMobius.pdf