MINIMAL SPACE WITH NON-MINIMAL SQUARE

MAGDALENA FORYŚ-KRAWIEC

Abstract. We will continue the talk based on paper [2] by L. Snoha and V. Spitalsky on minimal spaces with non-minimal square, presenting the proof of the following theorem:

Theorem 1. *There exists a metric continuum X admitting a minimal homeomorphism, such that the product $X \times X$ does not admit any minimal continuous map.*

We have already defined a Slovak space X as the closure of the graph of a function $F : X_d \to [0,1]$, where $X_d = (C \times [0,1])_{(y,1)\sim(h(y),0)}$ is a generalized solenoid. We have also classified the path components of space X (α denotes the orbit of some arbitrarily chosen point $x_0 \in X_d$):

(1) $\mathcal{A} = \pi^{-1}(X_d \setminus \alpha)$ consisting of continuous injective images of real line,
(2) $\gamma = \pi^{-1}(\alpha)$ consisting of countably many components $\{C_n\}_{n\in\mathbb{Z}}$, where each component is a continuous injective image of the closed half-line;

and the path component of the product $X \times X$:

(1) $\alpha \times \beta$, $\alpha, \beta \in \mathcal{A}$,
(2) $C_m \times \alpha$, $m \in \mathbb{Z}$, $\alpha \in \mathcal{A}$,
(3) $\alpha \times C_m$, $m \in \mathbb{Z}$, $\alpha \in \mathcal{A}$,
(4) $C_m \times C_n$, $m, n \in \mathbb{Z}$.

We said that the only components dense in $X \times X$ are those of the first type. We also showed that for any continuous map $G : X \times X \to X \times X$ the set of components of first type is invariant and $\gamma \times \gamma$ is a fixed point of G.

In the continuation of the talk we show that any continuous function $G : X \times X \to X \times X$ has to be of the form $T^{a\times b}$ for some nonzero $a, b \in \mathbb{Z}$, where T is a minimal homeomorphism on X. Then we deduce that in such a case G cannot be minimal.

References

[1] T. Downarowicz, L. Snoha, D. Tywoniuk, *Minimal spaces with cyclic group of homeomorphisms*, J. Dynam. Diff. Eq. **29** (2017) no. 1, 243-257

[2] L. Snoha, V. Spitalsky *Minimal space with non-minimale square* arXiv:1803.06323