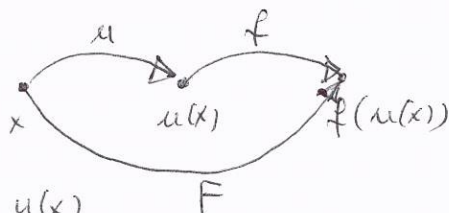


Różniczkowanie złożenie

① $f(u), u(x)$

$$F(x) := f(u(x))$$



$$\frac{dF}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

Tutaj u jest funkcją $u(x)$
- skrótowny zapis

Tutaj u jest argumentem funkcji $f(u)$

$$\frac{dF}{dx}(x) = \frac{df}{du}(u(x)) \cdot \frac{du}{dx}(x)$$

$$F'(x) = f'(u(x)) \cdot u'(x)$$

Żeby lepiej zrozumieć,
można pisać $f(p)$ zamiast $f(u)$.

Wtedy

$$\frac{dF}{dx}(x) = \frac{df}{dp}(u(x)) \cdot \frac{du}{dx}(x)$$

R12.

$$f(u) = 2u^3 + 1, \quad u \in \mathbb{R}$$

$$u(x) = \sin x, \quad x \in \mathbb{R}$$

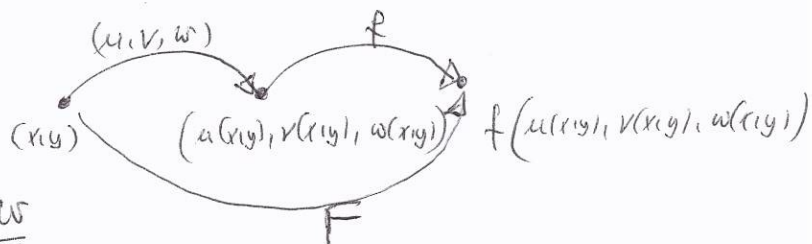
$$\bullet F(x) = f(u(x)) = f(\sin x) = 2 \sin^3 x + 1$$

$$f'(u) = 6u^2, \quad u'(x) = \cos x$$

$$\bullet F'(x) = \underbrace{6 \sin^2 x}_{f'(\sin x)} \cdot \underbrace{\cos x}_{u'(x)}$$

② $f(u, v, w), u(x, y), v(x, y), w(x, y)$

$$F(x, y) := f(u(x, y), v(x, y), w(x, y))$$



$$\frac{\partial F}{\partial x} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial x} + \frac{\partial f}{\partial w} \cdot \frac{\partial w}{\partial x}$$

$$\frac{\partial F}{\partial y} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial y} + \frac{\partial f}{\partial w} \cdot \frac{\partial w}{\partial y}$$

> skrótny zapis

$$\begin{aligned} \frac{\partial F}{\partial x}(x, y) &= \frac{\partial f}{\partial u}(u(x, y), v(x, y), w(x, y)) \cdot \frac{\partial u}{\partial x}(x, y) \\ &+ \frac{\partial f}{\partial v}(u(x, y), v(x, y), w(x, y)) \cdot \frac{\partial v}{\partial x}(x, y) \\ &+ \frac{\partial f}{\partial w}(u(x, y), v(x, y), w(x, y)) \cdot \frac{\partial w}{\partial x}(x, y) \end{aligned}$$

Żeby lepiej zrozumieć,
można pisać
 $f(p, q, r)$ zamiast
 $f(u, v, w)$.

$$\boxed{\begin{aligned}\frac{\partial F}{\partial y}(x,y) &= \frac{\partial f}{\partial u}(u(x,y), v(x,y), w(x,y)) \cdot \frac{\partial u}{\partial y}(x,y) \\ &+ \frac{\partial f}{\partial v}(u(x,y), v(x,y), w(x,y)) \cdot \frac{\partial v}{\partial y}(x,y) \\ &+ \frac{\partial f}{\partial w}(u(x,y), v(x,y), w(x,y)) \cdot \frac{\partial w}{\partial y}(x,y)\end{aligned}}$$

Q12

$$f(u, v, w) = u^2 v + v^3 + w^4, \quad u, v, w \in \mathbb{R}$$

$$u(x,y) = \sin xy, \quad v(x,y) = 2x + 3y, \quad w(x,y) = x^2 + y^2, \quad x, y \in \mathbb{R}$$

$$\bullet F(x,y) = f(u(x,y), v(x,y), w(x,y)) = (\sin^2 xy) \cdot (2x + 3y) + (2x + 3y)^3 + (x^2 + y^2)^4$$

$$\frac{\partial f}{\partial u}(u, v, w) = 2uv, \quad \frac{\partial f}{\partial v}(u, v, w) = u^2 + 3v^2, \quad \frac{\partial f}{\partial w}(u, v, w) = 4w^3$$

$$\frac{\partial u}{\partial x}(x,y) = y \cos xy, \quad \frac{\partial v}{\partial x}(x,y) = 2, \quad \frac{\partial w}{\partial x}(x,y) = 2x$$

$$\frac{\partial u}{\partial y}(x,y) = x \cos xy, \quad \frac{\partial v}{\partial y}(x,y) = 3, \quad \frac{\partial w}{\partial y}(x,y) = 2y$$

$$\begin{aligned}\bullet \frac{\partial F}{\partial x}(x,y) &= \underbrace{2(\sin xy)(2x+3y)}_{\frac{\partial f}{\partial u}(\sin xy, 2x+3y, x^2+y^2)} \cdot \underbrace{y \cos xy}_{\frac{\partial u}{\partial x}(x,y)} \\ &+ \underbrace{(u^2 xy + 3(2x+3y)^2)}_{\frac{\partial f}{\partial v}(\sin xy, 2x+3y, x^2+y^2)} \cdot \underbrace{2}_{\frac{\partial v}{\partial x}(x,y)} \\ &+ \underbrace{4(x^2+y^2)^3}_{\frac{\partial f}{\partial w}(\sin xy, 2x+3y, x^2+y^2)} \cdot \underbrace{2x}_{\frac{\partial w}{\partial x}(x,y)}\end{aligned}$$

$$\begin{aligned}\bullet \frac{\partial F}{\partial y}(x,y) &= \underbrace{2(\sin xy)(2x+3y)}_{\frac{\partial f}{\partial u}(\sin xy, 2x+3y, x^2+y^2)} \cdot \underbrace{x \cos xy}_{\frac{\partial u}{\partial y}(x,y)} \\ &+ \underbrace{(u^2 xy + 3(2x+3y)^2)}_{\frac{\partial f}{\partial v}(\sin xy, 2x+3y, x^2+y^2)} \cdot \underbrace{3}_{\frac{\partial v}{\partial y}(x,y)} \\ &+ \underbrace{4(x^2+y^2)^3}_{\frac{\partial f}{\partial w}(\sin xy, 2x+3y, x^2+y^2)} \cdot \underbrace{2y}_{\frac{\partial w}{\partial y}(x,y)}\end{aligned}$$