1) $x' = \dfrac{t^2x^2+1}{2t^2}$

- $x' = \frac{1}{2}x^2 + \frac{1}{2t^2}$

$x_1 = \frac{a}{t}$, $x_1' = -\frac{a}{t^2}$

$-\frac{a}{t^2} = \frac{1}{2}\frac{a^2}{t^2} + \frac{1}{2t^2} \quad / \cdot 2t^2$

$-2a = a^2 + 1$

$a^2 + 2a + 1 = 0$

$(a+1)^2 = 0$

$a = -1$

$\underline{x_1 = -\frac{1}{t}}$ 1

$x = -\frac{1}{t} + \frac{1}{y}$

$y' = -\left(2 \cdot \frac{1}{2} \cdot \left(-\frac{1}{t}\right)\right) y - \frac{1}{2}$

- $y' = \frac{1}{t}y - \frac{1}{2}$ 1

$y_h = C e^{\int \frac{dt}{t}} = C e^{\ln|t|} = C|t|$

$\underline{y_h = Ct}$

$y_p = C(t)\,t$

$C'(t)\,t = -\frac{1}{2}$

$C'(t) = -\frac{1}{2t}$

$C(t) = -\frac{1}{2}\ln|t|$

$y_p = -\frac{1}{2}t\ln|t|$

$\underline{y = Ct - \frac{1}{2}t\ln|t|}$ 2

$$\boxed{x = -\frac{1}{t} + \frac{1}{Ct - \frac{1}{2}t\ln|t|}}$$ 1

☒

2. $\underbrace{(x+\sin t\cos^2 tx)}_{P}\,dt+\underbrace{(t+\sin x\cos^2 tx)}_{Q}\,dx=0$

$w=tx$

$$\frac{Q_t-P_x}{w_xP-w_tQ}=\frac{\cancel{1}-2x\sin x\cos tx\sin tx-\cancel{1}+2t\sin t\cos tx\sin tx}{t(x+\sin t\cos^2 tx)-x(t+\sin x\cos^2 tx)}=$$

$$=\frac{2\cos tx\sin tx\,(t\sin t-x\sin x)}{t\sin t\cos^2 tx-x\sin x\cos^2 tx}=\frac{2\cos tx\sin tx\,\cancel{(t\sin t-x\sin x)}}{\cos^2 tx\,\cancel{(t\sin t-x\sin x)}}$$

$$=\frac{2\sin tx}{\cos tx}=2\,tg\,tx=2\,tg\,w$$

$$\mu=C\,e^{2\int tg\,w\,dw}=Ce^{-2\ln|\cos w|},\quad \boxed{\mu=\frac{1}{\cos^2 tx}}$$

~~2~~ 1

$$\left(\frac{x}{\cos^2 tx} + \sin t\right) dt + \left(\frac{t}{\cos^2 tx} + \sin x\right) dx = 0$$

$$\begin{cases} \frac{\partial F}{\partial t} = \frac{x}{\cos^2 tx} + \sin t \rightarrow F = \int\left(\frac{x}{\cos^2 tx} + \sin t\right) dt = -\cos t + \int \frac{du}{\cos^2 u} \\ \frac{\partial F}{\partial x} = \frac{t}{\cos^2 tx} + \sin x \end{cases}$$

$$\left\{\begin{matrix} u = tx \\ du = x\,dt \end{matrix}\right\} = \operatorname{tg} u - \cos t + \varphi(x) = \operatorname{tg} tx - \cos t + \varphi(x)$$

$$\int \frac{du}{\cos^2 u} \underset{w = \operatorname{tg} u}{=} \int \frac{1}{\left(\frac{1}{1+t^2}\right)} \cdot \frac{dw}{1+t^2} = \int dw = w + C = \operatorname{tg} u + C$$

Albo skorzystać z gotowego wzoru na tg całkę

$$\frac{t}{\cos^2 tx} + \varphi'(x) = \frac{t}{\cos^2 tx} + \sin x$$

$$\varphi(x) = -\cos x$$

$$F = \operatorname{tg} tx - \cos t - \cos x \quad \cancel{2} \quad 3$$

$$\boxed{\operatorname{tg} tx - \cos t - \cos x = C} \quad 1$$

Jeśli zostawi F i nie ~~przyrówna~~ przyrówna do C, to 1 pkt!

③ $2x\,x'' - 3(x')^2 = 4x^2$

$y = x'$, $y = y(x)$, $x'' = y'x' = yy'$

$2x\,yy' - 3y^2 = 4x^2$

$x \neq 0$ (Jeśli nie rozważy $x = 0$ nic nie ucieka)

$$yy' = \frac{3y^2}{2x} + \frac{4x^2}{2x}$$

$yy' = \frac{3}{2x} \cdot y^2 + 2x$ — Gdyby podzielić przez y, to byłoby równanie Bernoulliego

$z = y^2$, $z = z(x)$, $z' = 2yy'$

$\frac{1}{2} z' = \frac{3}{2x} z + 2x$

• $z' = \frac{3}{x} z + 4x$ 1

$z_j = C e^{\int \frac{3}{x} dx} = Cx^3$

$z_n = C(x)\, x^3$

$C'(x)\, x^3 = 4x$

$C'(x) = \frac{4}{x^2}$, $C(x) = -\frac{4}{x}$

$z = Cx^3 - 4x^2$

$y^2 = Cx^3 - 4x^2$

$y = \pm 2\sqrt{Cx^3 - \cancel{4}x^2}$

•• $x' = \pm 2\sqrt{Cx^3 - \cancel{4}x^2}$ 2

Rozważmy z „+". Z „−" analogicznie

$\int \frac{dx}{\sqrt{Cx^3 - \cancel{4}x^2}} = 2\int dt$

$\int \frac{dx}{|x| \cdot \sqrt{Cx - 1}} = 2t$

Rozważmy $x > 0$, $C \neq 0$ – nie czepiamy się ...

$\int \frac{dx}{x \cdot \sqrt{Cx-1}} = \dots$

$u = \sqrt{Cx-1}$, $Cx = u^2+1$, $x = \frac{1}{C}(u^2+1)$

$du = \frac{C}{2\sqrt{Cx-1}}\, dx$

$\dots \int \frac{2\sqrt{Cx-1}}{C} \cdot \frac{du}{x\sqrt{Cx-1}} = \frac{2}{\cancel{C}} \int \frac{\cancel{C}\, du}{u^2+1} = 2 \operatorname{arctg} u$

$= 2 \operatorname{arctg} \sqrt{Cx-1} + D$

$$\boxed{2 \operatorname{arctg} \sqrt{Cx-1} + D = 2t}$$ 2

$\boxtimes$

(4) $F(t, x, x', x'') = 0$

$x = e^y \quad , y = y(t)$

$x' = e^y y' \quad , \quad x'' = e^y (y')^2 + e^y y''$

$F(t, e^y, e^y y', e^y (y')^2 + e^y y'') = 0$

$(e^y)^m \cdot F(t, 1, y', (y')^2 + y'') = 0$

$F(t, 1, y', (y')^2 + y'') = 0$ (4)

$$\boxed{G(t, v_1, v_2) := F(t, 1, v_1, v_1^2 + v_2)}$$ (1)