Rozwiązania, Zestaw ~~10~~ 11: Liczby zespolone

1. a) $22+7i$

b) $-\frac{3}{29}-\frac{7}{29}i$

c) $-\frac{1}{2}+\frac{3}{2}i$

d) $\sqrt{12-5i}=\left\{\frac{5}{\sqrt{2}}-\frac{1}{\sqrt{2}}i,\ -\frac{5}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right\}$

e) $\sqrt{3+4i}=\{2+i,\ -2-i\}$

f) $\sqrt{2i}=\{1+i,\ -1-i\}$

2. a) $w_0=1$
$w_1=-1$

b) $w_0=1$
$w_1=i$
$w_2=-1$
$w_3=-i$

c) $w_0=\frac{\sqrt{3}}{2}+\frac{1}{2}i$
$w_1=-\frac{\sqrt{3}}{2}+\frac{1}{2}i$
$w_2=-i$

d) $w_0=\sqrt[10]{2}\left(\cos\frac{\pi}{20}+i\sin\frac{\pi}{20}\right)$
$w_1=\sqrt[10]{2}\left(\cos\frac{9\pi}{20}+i\sin\frac{9\pi}{20}\right)$
$w_2=\sqrt[10]{2}\left(\cos\frac{17\pi}{20}+i\sin\frac{17\pi}{20}\right)$
$w_3=\sqrt[10]{2}\left(\cos\frac{25\pi}{20}+i\sin\frac{25\pi}{20}\right)=\sqrt[10]{2}\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)$
$w_4=\sqrt[10]{2}\left(\cos\frac{33\pi}{20}+i\sin\frac{33\pi}{20}\right)$

e) $w_0=\sqrt[14]{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)=\sqrt[14]{2}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)$
$w_1=\sqrt[14]{2}\left(\cos\frac{15\pi}{28}+i\sin\frac{15\pi}{28}\right)$
$w_2=\sqrt[14]{2}\left(\cos\frac{23\pi}{28}+i\sin\frac{23\pi}{28}\right)$
$w_3=\sqrt[14]{2}\left(\cos\frac{31\pi}{28}+i\sin\frac{31\pi}{28}\right)$

f) $w_0 = \sqrt[4]{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

$w_1 = \sqrt[4]{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$

$w_2 = \sqrt[4]{2}\left(\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}\right)$

$w_3 = \sqrt[4]{2}\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right)$

g) $w_0 = \sqrt{3} + i$

$w_1 = -1 + \sqrt{3}i$

$w_2 = -\sqrt{3} - i$

$w_3 = 1 - \sqrt{3}i$

h) $w_0 = \frac{1}{\sqrt[12]{2}}\left(\cos\frac{19\pi}{72} + i\sin\frac{19\pi}{72}\right)$

$w_1 = \frac{1}{\sqrt[12]{2}}\left(\cos\frac{43\pi}{72} + i\sin\frac{43\pi}{72}\right)$

$w_2 = \frac{1}{\sqrt[12]{2}}\left(\cos\frac{67\pi}{72} + i\sin\frac{67\pi}{72}\right)$

$w_3 = \frac{1}{\sqrt[12]{2}}\left(\cos\frac{91\pi}{72} + i\sin\frac{91\pi}{72}\right)$

$w_4 = \frac{1}{\sqrt[12]{2}}\left(\cos\frac{115\pi}{72} + i\sin\frac{115\pi}{72}\right)$

$w_5 = \frac{1}{\sqrt[12]{2}}\left(\cos\frac{139\pi}{72} + i\sin\frac{139\pi}{72}\right)$

$\cancel{w_6 = \frac{1}{\sqrt[12]{2}}}$

3. a) $-8 + 8\sqrt{3}i$

b) $2^9\sqrt{3} - 2^9 i$

c) $-2^7 + 2^7\sqrt{3}i$

d) 2^9

4. a) $z_1 = 1 - 2i$, $z_2 = 1 + 2i$

b) $z_1 = -2 - i$, $z_2 = -2 + i$

c) $z_1 = 3 - \sqrt{2}i$, $z_2 = 3 + \sqrt{2}i$

d) $z_1 = 1$, $z_2 = -1$, $z_3 = 2i$, $z_4 = -2i$

e) $z_1 = 2$, $z_2 = 1 - i$, $z_3 = 1 + i$

f) $z_1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$, $z_2 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

g) $z_1 = -\frac{1}{5} + \frac{3}{5}i$, $z_2 = -\frac{4}{5} + \frac{7}{5}i$

h) $z_1 = 4$, $z_2 = i$, $z_3 = -i$

5. a) $z \in \sqrt[5]{-1}$

$z_0 = w_0 = \cos\frac{\pi}{5} + i\sin\frac{\pi}{5}$

$z_1 = w_1 = \cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}$

$z_2 = w_2 = -1$

$z_3 = w_3 = \cos\frac{7\pi}{5} + i\sin\frac{7\pi}{5}$

$z_4 = w_4 = \cos\frac{9\pi}{5} + i\sin\frac{9\pi}{5}$

b) $z \in \sqrt[4]{-i}$

$z_0 = w_0 = \cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}$

$z_1 = w_1 = \cos\frac{7\pi}{8} + i\sin\frac{7\pi}{8}$

$z_2 = w_2 = \cos\frac{11\pi}{8} + i\sin\frac{11\pi}{8}$

$z_3 = w_3 = \cos\frac{15\pi}{8} + i\sin\frac{15\pi}{8}$

c) $z + i \in \sqrt[5]{i}$

$w_0 = \cos\frac{\pi}{10} + i\sin\frac{\pi}{10}$

$w_1 = i$

$w_2 = \cos\frac{9\pi}{10} + i\sin\frac{9\pi}{10}$

$w_3 = \cos\frac{13\pi}{10} + i\sin\frac{13\pi}{10}$

$w_4 = \cos\frac{17\pi}{10} + i\sin\frac{17\pi}{10}$

Stąd: $z_0 = w_0 - i$

$z_1 = w_1 - i = 0$

$z_2 = w_2 - i$

$z_3 = w_3 - i$

$z_4 = w_4 - i$

d) $z^4 \in \sqrt[4]{-i}$

z_0, z_1, z_2, z_3 – jak w b)

e) $z - 1 \in \sqrt[3]{i}$

$w_0 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

$w_1 = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

$w_2 = -i$

Stąd: $z_0 = w_0 + 1 = \left(1 + \frac{\sqrt{3}}{2}\right) + \frac{1}{2}i$

$z_1 = w_1 + 1 = \left(1 - \frac{\sqrt{3}}{2}\right) + \frac{1}{2}i$

$z_2 = w_2 + 1 = 1 - i$

f) $z_1 = \frac{1}{2} + \frac{2-\sqrt{3}}{2}i, \quad z_2 = -\frac{1}{2} - \frac{2+\sqrt{3}}{2}i$

6. a) Prosta $x = 3$

b) Prosta $y = x - 2$

c) Parabola $y^2 = 4x$

d) $y = \frac{1}{\sqrt{3}}|x|$

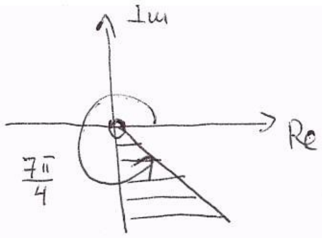
e) okręgi $x^2 + y^2 = 16$

f) okręgi $(x-2)^2 + (y+3)^2 = 81$

i) $x^2 + (y-2)^2 \geq 6$

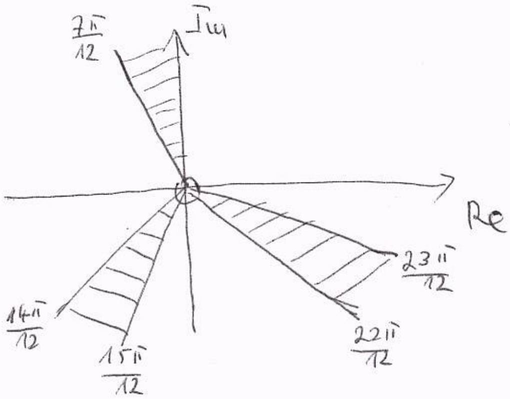
g), h) „wiatraki"

g)



$$-\frac{\pi}{2} + 2\pi(k-l) \le \arg z \le -\frac{\pi}{4} + 2\pi(k-l)$$
$$z \neq 0$$

h)



$$-\frac{\pi}{6} - \frac{2}{3}\pi(k-3l) \le \arg z \le -\frac{\pi}{12} + \frac{2}{3}\pi(k-3l)$$
$$z \neq 0$$

7*, 8* – samodzielnie!