① $f(x,y,z) = x^2+y^2+z^2-xy+y-2z$ $\qquad D=\mathbb{R}^3$

$$\frac{\partial f}{\partial x} = 2x-y$$
$$\frac{\partial f}{\partial y} = 2y-x+1$$
$$\frac{\partial f}{\partial z} = 2z-2$$

$$\begin{cases} 2x-y=0 \\ 2y-x+1=0 \\ 2z-2=0 \end{cases}$$

$y=2x$ $\qquad y=-\frac{2}{3}$

$4x-x+1=0,\quad x=-\frac{1}{3}$

$z=1$ $\qquad \underline{P=\left(-\frac{1}{3},-\frac{2}{3},1\right)}$

$$M(x,y,z)=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad M(P)=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

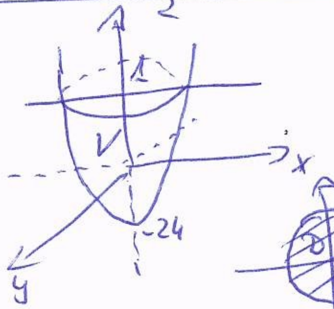
$\delta_1 = 2>0$

$\delta_2 = 3>0$

$\delta_3 = 6>0$

$\boxed{P=\left(-\frac{1}{3},-\frac{2}{3},1\right) \text{ min. lok.}}$

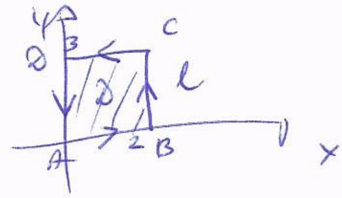
② Objętość: $z=x^2+y^2-24$, $z=1$



$$|V|=\iint_D \left[1-(x^2+y^2-24)\right]dx\,dy = \int_0^{2\pi}d\varphi\int_0^5(-r^2+25)\,r\,dr =$$
$$=\int_0^{2\pi}\left[-\frac{1}{4}r^4+\frac{25}{2}r^2\right]_0^5 d\varphi = \int_0^{2\pi}\left(-\frac{625}{4}+\frac{625}{2}\right)d\varphi =$$
$$=\left(-\frac{625}{4}+\frac{625}{2}\right)\int_0^{2\pi}d\varphi = 2\pi\cdot\left(-\frac{625}{4}+\frac{625}{2}\right) = \boxed{\frac{625}{2}\pi}$$

③ $\oint_l \underbrace{(x+y)^2}_{P}dx + \underbrace{(x-y)^2}_{Q}dy =$

l: brzeg prostokąta zorientowany dodatnio, o wierzchołkach:
$A=(0,0)$, $B=(2,0)$, $C=(2,3)$, $D(0,3)$



$$=\iint_D\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dx\,dy = \iint_D\left[2(x-y)-2(x+y)\right]dx\,dy =$$
$$=-4\iint_D y\,dx\,dy = -4\int_0^3 dy\int_0^2 y\,dx = -4\int_0^3 y\,[x]_0^2 =$$
$$=-4\cdot 2\int_0^3 y\,dy = -4\left[y^2\right]_0^3 = \boxed{-36}$$

④ $x' = tx+tx^2$

$x'=t(x+x^2)$ – r.o. zm. rozdzielonych

$x'=t\,x(x+1)$

1° $\boxed{x(t)\equiv 0,\quad x(t)\equiv -1}$

2° $x\neq 0,\ x\neq -1$

$$\int\frac{dx}{x(x+1)} = \int t\,dt$$

$$\frac{1}{x(x+1)} = \frac{A}{x}+\frac{B}{x+1}$$
$$1 = A(x+1)+Bx$$
$$1 = (A+B)x+A$$
$$\begin{cases} A+B=0 & B=-1 \\ A=1 \end{cases}$$

$$\int\frac{dx}{x(x+1)} = \int\frac{dx}{x}-\int\frac{dx}{x+1} = \ln|x|-\ln|x+1|+C$$

$$\boxed{\ln|x|-\ln|x+1| = \frac{1}{2}t^2+C}$$

(To jest też równanie Bernoulliego)

⑤ $x''-x'-2x=0$

$\lambda^2-\lambda-2=0$

$\Delta=9$

$\lambda_1=\frac{1-3}{2}=-1,\quad \lambda_2=\frac{1+3}{2}=2$

$$\boxed{x=C_1e^{-t}+C_2e^{2t}}$$