1) $f(x,y,z) = 2y^2 - xy + 2yz - x + x^3 + z^2$ $\quad D_f = \mathbb{R}^2$

$\frac{\partial f}{\partial x} = -y - 1 + 3x^2$

$\frac{\partial f}{\partial y} = 4y - x + 2z$

$\frac{\partial f}{\partial z} = 2y + 2z$

$$\begin{cases} -y - 1 + 3x^2 = 0 \\ 4y - x + 2z = 0 \\ 2y + 2z = 0 \end{cases}$$

$3x^2 - \frac{1}{2}x - 1 = 0$

$4y = x - 2z = x + 2y$, $y = \frac{1}{2}x$

$z = -y$

$3x^2 - \frac{1}{2}x - 1 = 0$

$6x^2 - x - 2 = 0$

$\Delta = 1 + 48 = 49$

$x = \frac{1 \pm 7}{12}$

$x_1 = -\frac{1}{2}$ $\quad x_2 = \frac{8}{12} = \frac{2}{3}$

$y_1 = -\frac{1}{4}$ $\quad y_2 = \frac{1}{3}$

$z_1 = \frac{1}{4}$ $\quad z_2 = -\frac{1}{3}$

$P_1 = (-\frac{1}{2}, -\frac{1}{4}, \frac{1}{4})$

$P_2 = (\frac{2}{3}, \frac{1}{3}, -\frac{1}{3})$

5

$$H(x,y,z) = \begin{bmatrix} 6x & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

$$H(P_1) = \begin{bmatrix} -3 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

$D_1 = -3 < 0$

$D_2 = -13 < 0$

$D_3 = -24 + 12 - 2 = -14 < 0$

$\boxed{\text{Brak w } P_1 \text{ ekstremum}}$

5

$$H(P_2) = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

$D_1 = 4 > 0$

$D_2 = 15 > 0$

$D_3 = 32 - 16 - 2 = 14 > 0$

$\boxed{\text{min. lok. w } P_2}$

---

2) $x^3 + 2xy + y^2 = 0$, $x \neq 0$

$f(x,y) = x^3 + 2xy + y^2$

$\frac{\partial f}{\partial x} = 3x^2 + 2y$

$\frac{\partial f}{\partial y} = 2x + 2y$

$$\begin{cases} x^3 + 2xy + y^2 = 0 \\ 3x^2 + 2y = 0 \end{cases} \Rightarrow y = -\frac{3}{2}x^2$$

$x^3 - 3x^3 + \frac{9}{4}x^4 = 0$

$-2x^3 + \frac{9}{4}x^4 = 0$

$-8x^3 + 9x^4 = 0$

$x^3(9x - 8) = 0$

$\underset{\text{Odpada}}{x = 0}$ $\vee$ $x = \frac{8}{9}$

$y = -\frac{\cancel{3}}{\cancel{2}} \cdot \frac{\overset{32}{\cancel{64}}}{\underset{27}{\cancel{81}}} = -\frac{32}{27}$

5

$\frac{\partial^2 f}{\partial x^2} = 6x$

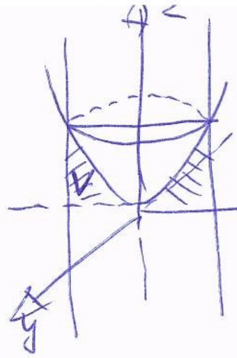
$y''(x) = -\frac{\frac{\partial^2 f}{\partial x^2}(x, y(x))}{\frac{\partial f}{\partial y}(x, y(x))} = \frac{-\overset{3}{\cancel{6}}x}{\cancel{2}(x+y)} = -\frac{3x}{x+y}$

$y''(\frac{8}{9}) = -\frac{3 \cdot \frac{8}{9}}{\frac{8}{9} - \frac{32}{27}} > 0$

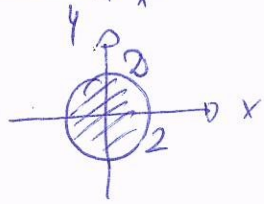
$\boxed{\text{min. lok. w } x_0 = \frac{8}{9}}$

5

(3) $z = x^2+y^2, \quad x^2+y^2=4, \quad z=0$



2



2

$$|V| = \iint_D (x^2+y^2)\,dx\,dy = \int_0^{2\pi} d\varphi \int_0^2 r^3\,dr =$$

$$= \frac{1}{4}\int_0^{2\pi} r^4 \Big|_0^2 \,d\varphi = \frac{1}{4}\cdot 16 \int_0^{2\pi} d\varphi =$$

$$= 4\,\varphi\Big|_0^{2\pi} = \boxed{8\pi}$$

6

---

(4) $$\int_{(-1,1)}^{(2,3)} \underbrace{3x^2y}_{P}\,dx + \underbrace{x^3}_{Q}\,dy = x^3y\Big|_{(-1,1)}^{(2,3)} = 24-(-1) = \boxed{25}$$

4

$$\frac{\partial Q}{\partial x} = 3x^2 = \frac{\partial P}{\partial y}$$

$$\begin{cases} \dfrac{\partial F}{\partial x} = 3x^2y \implies F = \int 3x^2y\,dx = x^3y + \varphi(y) \\ \dfrac{\partial F}{\partial y} = x^3 \end{cases}$$

$$\cancel{x^3} + \varphi'(y) = \cancel{x^3}$$

$$\varphi'(y) = 0$$

$$\varphi(y) = 0$$

$$\underline{F = x^3y}$$

6

---

(5) $$x' = (t^2+t+2)\cdot(x^2+x)$$

5

$$\int \frac{dx}{x^2+x} = \int (t^2+t+2)\,dt$$

$$\int \frac{dx}{x^2+x} = \int \frac{dx}{x(x+1)} = \int \frac{dx}{x} - \int \frac{dx}{x+1} = \ln|x| - \ln|x+1| + C$$

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$\boxed{\ln|x| - \ln|x+1| = \frac{1}{3}t^3 + \frac{1}{2}t^2 + 2t + C}$$

5

$$1 = A(x+1) + Bx$$

$$\begin{cases} A+B=0 \\ A=1 \end{cases}, \quad B=-1$$