① $f(x) = \sqrt{2-|x^2+x|}$ – dziedzina

1) $2 - |x^2+x| \geq 0$

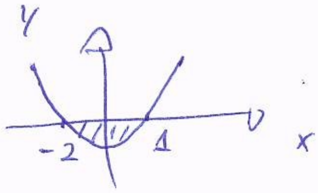
$|x^2+x| \leq 2$

$x^2+x \geq -2 \quad \wedge \quad x^2+x \leq 2$

$x^2+x+2 \geq 0 \qquad x^2+x-2 \leq 0$

$\Delta = -7 < 0 \qquad \Delta = 9$

$\underline{x \in \mathbb{R}} \qquad x_1 = \frac{-1-3}{2} = -2, \quad x_2 = \frac{-1+3}{2} = 1$



$\underline{x \in \langle -2, 1 \rangle}$

$\boxed{D_f = \langle -2, 1 \rangle}$

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② a) $\lim\limits_{n\to\infty} \left(\frac{n+1}{n+2}\right)^n \overset{1^\infty}{=} \lim\limits_{n\to\infty} \left(\frac{(n+2)-1}{n+2}\right)^n = \lim\limits_{n\to\infty} \left[\left(1 + \frac{1}{-(n+2)}\right)^{-(n+2)}\right]^{\frac{n}{-(n+2)}} = \boxed{e^{-1}}$

Bo: $\lim\limits_{n\to\infty} [-(n+2)] = -\infty, \quad \lim\limits_{n\to\infty} \frac{n}{-(n+2)} = -1$

b) $\lim\limits_{x\to 0} \frac{x^2+\cos^3 x - 1}{\cos 4x - 1} \overset{\frac{0}{0}}{\underset{H}{=}} \lim \frac{2x - 3\cos^2 x \sin x}{-4\sin 4x} \overset{\frac{0}{0}}{\underset{H}{=}} \lim\limits_{x\to 0} \frac{2-3(-2\cos x \sin^2 x + \cos^3 x)}{-16\cos 4x} = \boxed{\frac{1}{16}}$

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③ $f(x) = e^{2x} \cdot \left(3 + \frac{2}{x}\right) \qquad D_f = \mathbb{R} \setminus \{0\}$

$f'(x) = 2e^{2x}\left(3+\frac{2}{x}\right) + e^{2x}\left(-\frac{2}{x^2}\right) = e^{2x}\left(6+\frac{4}{x}-\frac{2}{x^2}\right) =$

$= \frac{6x^2+4x-2}{x^2} \cdot e^{2x}$

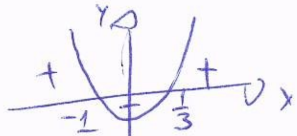
$6x^2+4x-2 = 0$

$3x^2+2x-1 = 0$

$\Delta = 4+12 = 16$

$x_1 = \frac{-2-4}{6} = -1$

$x_2 = \frac{-2+4}{6} = \frac{1}{3}$



$x \in (-\infty, -1),\ f'(x) > 0,$ $f(x)\ \uparrow$ | $x_1 = -1$ max. lok.

$x \in (-1, 0),\ f'(x) < 0,$ $f(x)\ \downarrow$

$x \in (0, \frac{1}{3}),\ f'(x) < 0,$ $f(x)\ \downarrow$ | $x_2 = \frac{1}{3}$ min. lok.

$x \in (\frac{1}{3}, \infty),\ f'(x) > 0,$ $f(x)\ \uparrow$

Bo $e^{2x} > 0,\ x^2 > 0.$ (dla $x \neq 0$)

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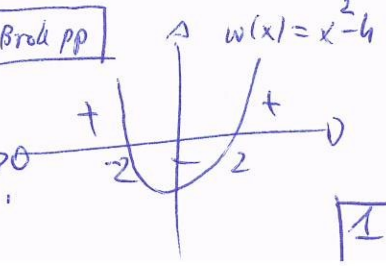
④ $f(x) = \frac{1}{x^2-4}, \qquad D_f = \mathbb{R} \setminus \{\pm 2\}$

$f'(x) = \frac{-2x}{(x^2-4)^2}, \qquad f''(x) = -2\frac{(x^2-4)^{\cancel{2}} - x\, 2\cancel{(x^2-4)}2x}{(x^2-4)^{\cancel{4}\,3}} = -2\frac{x^2-4-4x^2}{(x^2-4)^3} =$

$= 2 \cdot \frac{3x^2+4}{(x^2-4)^3}$

$3x^2+4 = 0$

$\boxed{x \in \emptyset, \text{ brak pp}}$



Bo $3x^2+4 > 0$ dla $x \in \mathbb{R}$.

$x \in (-\infty, -2),\ f''(x) > 0,$ $f(x)\ \cup$

$x \in (-2, 2),\ f''(x) < 0,$ $f(x)\ \cap$

$x \in (2, \infty),\ f''(x) > 0,$ $f(x)\ \cup$

5) a) $\int \frac{dx}{\sqrt{3x+2}+1} = \int \frac{2t\, dt}{3(t+1)} = \frac{2}{3}\int \frac{(t+1)-1}{t+1}\, dt$

$t = \sqrt{3x+2}$

$dt = \frac{3}{2\sqrt{3x+2}}\, dx$

$= \frac{2}{3}\left(\int dt - \int \frac{dt}{t+1}\right) = \frac{2}{3}t - \frac{2}{3}\ln|t+1| + C =$

$= \boxed{\frac{2}{3}\sqrt{3x+2} - \frac{2}{3}\ln\left|\sqrt{3x+2}+1\right| + C}$

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b) $\int x e^{-2x}\, dx = \left\{ \begin{array}{l|l} u = x & u' = 1 \\ v' = e^{-2x} & v = -\frac{1}{2}e^{-2x} \end{array} \right\} = -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\, dx =$

$= \boxed{-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C}$
