1) $f(x) = \log\left(\log_{\frac{1}{3}}(x^2-1)\right)$

1) $x^2-1>0$

$x^2>1$

$x<-1 \vee x>1$ (2)

2) $\log_{\frac{1}{3}}(x^2-1)>0$ 2- zoi.

$\log_{\frac{1}{3}}(x^2-1)>\log_{\frac{1}{3}}1$

$x^2-1<1$

$x^2<2$

$x\in(-\sqrt{2},\sqrt{2})$ (4)

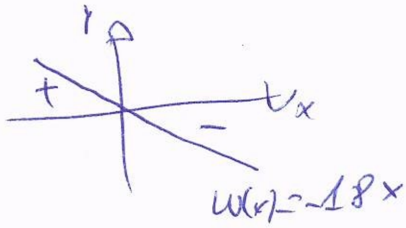
$\boxed{D_f=(-\sqrt{2},-1)\cup(1,\sqrt{2})}$ (2)

2) a) $\lim\limits_{n\to\infty}\left(\sqrt{n^2+4}-n\right) \overset{\infty-\infty}{=} \lim\limits_{n\to\infty}\dfrac{\cancel{n^2}+4-\cancel{n^2}}{\sqrt{n^2+4}+n} \overset{\frac{4}{\infty}}{=} \boxed{0}$ (5)

b) $\lim\limits_{x\to0}\dfrac{\cos4x-x-1}{x+\sin3x} \underset{H}{\overset{\frac{0}{0}}{=}} \lim\limits_{x\to0}\dfrac{-4\sin4x-1}{1+3\cos3x} = \boxed{-\frac{1}{4}}$ (5)

3) $f(x)=\dfrac{x^2}{x^2-9}$ $D_f=\mathbb{R}\setminus\{\pm3\}$

$f'(x)=\dfrac{2x(x^2-9)-x^2\cdot2x}{(x^2-9)^2}=\dfrac{-18x}{(x^2-9)^2}$



$-18x=0$

$x_0=0\in D_f$ (5)

$x\in(-\infty,-3)$, $f'(x)>0$, $f(x)\uparrow$

$x\in(-3,0)$, $f'(x)>0$, $f(x)\uparrow$ | $x_0=0$ maks. lk.

$x\in(0,3)$, $f'(x)<0$, $f(x)\downarrow$

$x\in(3,\infty)$, $f'(x)<0$, $f(x)\downarrow$ (5)

4) $f(x)=\frac{1}{20}x^5+\frac{1}{12}x^4+x+4$ $D_f=\mathbb{R}$

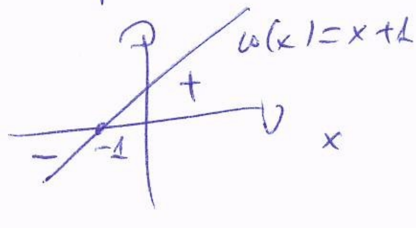
$f'(x)=\frac{1}{4}x^4+\frac{1}{3}x^3+1$

$f''(x)=x^3+x^2=x^2(x+1)$

$x^3+x=0$

$x^2(x+1)=0$

$x_1=0,\ x_2=-1\in D_f$ (5)



$x\in(-\infty,-1)$, $f''(x)<0$, $f(x)\cap$

$x\in(-1,0)$, $f''(x)>0$, $f(x)\cup$ | $x_0=-1$ PP

$x\in(0,\infty)$, $f''(x)>0$, $f(x)\cup$ (5)

5) a) $\int x\sin2x\,dx = \left\{\begin{matrix}u=x\\ v'=\sin2x\end{matrix}\middle|\begin{matrix}u'=1\\ v=-\frac{1}{2}\cos2x\end{matrix}\right\} = -\frac{1}{2}x\cos2x+\frac{1}{2}\int\cos2x\,dx$

$=\boxed{-\frac{1}{2}x\cos2x+\frac{1}{4}\sin2x+C}$ (5)

b) $\int\dfrac{3x}{3x+1}dx=\int\dfrac{(3x+1)-1}{3x+1}dx=\int dx-\frac{1}{3}\int\dfrac{3\,dx}{3x+1}=\boxed{x-\frac{1}{3}\ln|3x+1|+C}$ (5)