① $f(x) = \sqrt{1 - \log_2(x^2-4)}$

1) $x^2 - 4 > 0$

$x^2 > 4$

$x \in (-\infty, -2) \cup (2, \infty)$ 2

2) $1 - \log_2(x^2-4) \geq 0$ 2 za zbr.

$\log_2(x^2-4) \leq 1$

$\log_2(x^2-4) \leq \log_2 2$

$x^2 - 4 \leq 2$

$x^2 \leq 6$ $\quad x \in \langle -\sqrt{6}, \sqrt{6} \rangle$ 4


$\boxed{D_f = \langle -\sqrt{6}, -2) \cup (2, \sqrt{6} \rangle}$ 2

---

② a) $\lim_{n\to\infty} \sqrt[n]{3^n + 4^n} = \boxed{4}$

$4 = \sqrt[n]{4^n} \leq \sqrt[n]{3^n+4^n} \leq \sqrt[n]{2 \cdot 4^n} = 4 \cdot \sqrt[n]{2}$ 5

$\downarrow \quad \downarrow \quad \downarrow$

$4 \quad 4 \quad 4$

b) $\lim_{x\to 0} \dfrac{\cos^2 x - x - 1}{x + \sin^2 x} \overset{\frac{0}{0}}{\underset{H}{=}} \lim_{x\to 0} \dfrac{-2\cos x \overset{\to 0}{\sin x} - 1}{1 + 2\underset{\to 0}{\sin x}\cos x} = \boxed{-1}$ 5

---

③ $f(x) = \dfrac{1}{x^2+x}$

$x^2 + x = 0$

$x(x+1) = 0$

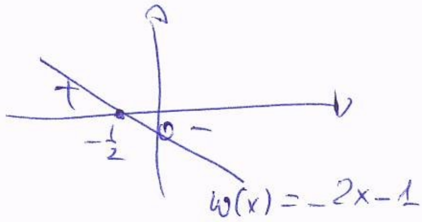
$x = 0 \lor x = -1$

$\underline{D_f = \mathbb{R} \setminus \{-1, 0\}}$

$f'(x) = \dfrac{-2x-1}{(x^2+x)^2}$

$-2x - 1 = 0$

$x_0 = -\frac{1}{2} \in D_f$ 5




| | | |
|---|---|---|
| $x \in (-\infty, -1)$, $f'(x) > 0$, | $f(x) \uparrow$ | |
| $x \in (-1, -\frac{1}{2})$, $f'(x) > 0$, | $f(x) \uparrow$ | $x_0 = -\frac{1}{2}$ |
| $x \in (-\frac{1}{2}, 0)$, $f'(x) < 0$, | $f(x) \downarrow$ | max. lok. |
| $x \in (0, \infty)$, $f'(x) < 0$, | $f(x) \downarrow$ | |

5

---

④ $f(x) = \frac{1}{30}x^6 - \frac{1}{20}x^5 + 2x + 3$ $\quad \underline{D_f = \mathbb{R}}$

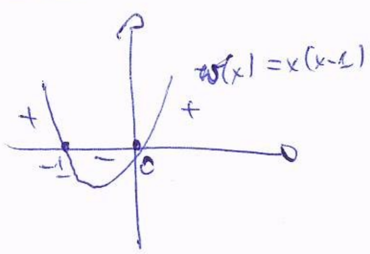
$f'(x) = \frac{1}{5}x^5 - \frac{1}{4}x^4 + 2$

$f''(x) = x^4 - x^3$

$x^4 - x^3 = 0$

$x^3(x-1) = 0$

$x_1 = 0,\ x_2 = 1 \in D_f$ 5




| | | |
|---|---|---|
| $x \in (-\infty, 0)$, $f''(x) > 0$, | $f(x) \cup$ | $x_1 = 0$ pp |
| $x \in (0, 1)$, $f''(x) < 0$, | $f(x) \cap$ | |
| $x \in (1, \infty)$, $f''(x) > 0$, | $f(x) \cup$ | $x_2 = 1$ pp |

5

---

⑤ a) $\int \dfrac{x}{\sqrt{x+1}}\,dx = \int \dfrac{t^2-1}{\cancel{t}} \cdot 2\cancel{t}\,dt = 2\int (t^2-1)\,dt =$

$t = \sqrt{x+1} \to x = t^2 - 1$ $\quad = \frac{2}{3}t^3 - 2t + C =$

$dt = \dfrac{1}{2\sqrt{x+1}}\,dx$ $\quad = \boxed{\frac{2}{3}(\sqrt{x+1})^3 - 2\sqrt{x+1} + C}$

5

b) $\int \dfrac{dx}{x^2 - 2x} = \int \dfrac{dx}{x(x-2)} =$

$= -\frac{1}{2}\int \dfrac{dx}{x} + \frac{1}{2}\int \dfrac{dx}{x-2} = \boxed{-\frac{1}{2}\ln|x| + \frac{1}{2}\ln|x-2| + C}$

5

$\dfrac{1}{x(x-2)} = \dfrac{A}{x} + \dfrac{B}{x-2}$ $\quad 1 = A(x-2) + Bx$

$A + B$

$\begin{cases} A + B = 0 \\ -2A = 1 \end{cases}$ $\quad A = -\frac{1}{2}$, $\quad B = \frac{1}{2}$