# Wydział Energetyki i Paliw AGH, Technologia Chemiczna
# Zadania z MATEMATYKI
# ZESTAW 3

## Ciągi liczbowe

1. Zbadać monotoniczność i ograniczoność ciągów:

   1.1. $a_n = \frac{4n+1}{2n+3}$,

   1.2. $a_n = \frac{3n+4}{6n+2}$,

   1.3. $a_n = \frac{n^2+2n+1}{n^2-3}$,

   1.4. $a_n = \frac{n^2+2}{n-1}$,

   1.5. $a_n = \frac{n}{2^n}$,

   1.6. $a_n = \frac{2^n}{n!}$,

   1.7. $a_n = \frac{3^n}{3^n+1}$,

   1.8. $a_n = n^{(-1)^n}$.

2. Korzystając z definicji granicy ciągu, udowodnić, że:

   2.1. $\lim\limits_{n\to\infty} \frac{2n}{3n+1} = \frac{2}{3}$,

   2.2. $\lim\limits_{n\to\infty} \frac{3n+2}{4n-1} = \frac{3}{4}$,

   2.3. $\lim\limits_{n\to\infty} \sqrt[3]{2n+1} = \infty$,

   2.4. $\lim\limits_{n\to\infty} (5 - 2^n) = -\infty$.

3. Obliczyć:

   3.1. $\lim\limits_{n\to\infty} \frac{7n^5+4n^3-1}{2n^5-9n^4+3n}$,

   3.2. $\lim\limits_{n\to\infty} \frac{-4n^6+3n^5-2n+1}{3n^4-7n^3+n}$,

   3.3. $\lim\limits_{n\to\infty} \frac{8n^4+4n^3-3}{2n^7+3n^4+8n^2+3}$,

   3.4. $\lim\limits_{n\to\infty} \sqrt[3]{\frac{64n^2+1}{n^2+4}}$,

   3.5. $\lim\limits_{n\to\infty} \frac{n+\sqrt{n}}{n-\sqrt{n}}$,

   3.6. $\lim\limits_{n\to\infty} \left(\sqrt{n^2+2} - n\right)$,

   3.7. $\lim\limits_{n\to\infty} n\left(\sqrt{n+1} - \sqrt{n}\right)$,

   3.8. $\lim\limits_{n\to\infty} \left(\sqrt{n^2+4n+1} - \sqrt{n^2+2}\right)$,

   3.9. $\lim\limits_{n\to\infty} \left(\sqrt[3]{n^3+3n} - n\right)$,

   3.10. $\lim\limits_{n\to\infty} \left(\sqrt{n+\sqrt{n+\sqrt{n}}} - \sqrt{n}\right)$,

   3.11. $\lim\limits_{n\to\infty} \frac{\sqrt{n^2+5}-n}{\sqrt{n^2+2}-n}$,

3.12. $\lim\limits_{n\to\infty} \frac{3^n+2^n}{4^n+5^n}$,

3.13. $\lim\limits_{n\to\infty} \frac{4\cdot 3^{n+1}+2\cdot 4^n}{5\cdot 2^n+4^{n+2}}$,

3.14. $\lim\limits_{n\to\infty} \frac{6^n-n}{n^2+2^n}$,

3.15. $\lim\limits_{n\to\infty} \frac{\sin n}{n^3+5}$,

3.16. $\lim\limits_{n\to\infty} \frac{\log_2 n^5}{\log_8 n}$,

3.17. $\lim\limits_{n\to\infty} n^2\left(\ln(n+1)-\ln n\right)$,

3.18. $\lim\limits_{n\to\infty} \frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$,

3.19. $\lim\limits_{n\to\infty} \sqrt[n]{2^n+3^n+4^n}$,

3.20. $\lim\limits_{n\to\infty} \sqrt[n]{5^n+7^n+1}$,

3.21. $\lim\limits_{n\to\infty} \sqrt[n]{2^n+n+\frac{1}{n}+\sin n}$,

3.22. $\lim\limits_{n\to\infty} \left(\frac{2n^2+1}{n^2+1}\right)^{\sqrt{n}+3}$,

3.23. $\lim\limits_{n\to\infty} \left(\frac{n+2}{n+5}\right)^{n+1}$,

3.24. $\lim\limits_{n\to\infty} \left(\frac{n^2+n-3}{n^2+4}\right)^{3n}$,

3.25. $\lim\limits_{n\to\infty} \left(\frac{n^2+3n+5}{n^2-2n+3}\right)^{n^2}$,

3.26. $\lim\limits_{n\to\infty} \frac{1+4+7+\ldots+(3n-2)}{n^2}$,

3.27. $\lim\limits_{n\to\infty} \frac{1^2+2^2+3^2+\ldots+n^2}{6n^3-n^2+2n+1}$,

3.28. $\lim\limits_{n\to\infty} \left(\frac{1+2+\ldots+n}{n+2}-\frac{n}{2}\right)$,

3.29. $\lim\limits_{n\to\infty} \frac{1+\frac{1}{2}+\frac{1}{4}+\ldots+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}+\ldots+\frac{1}{3^n}}$,

3.30. $\lim\limits_{n\to\infty} \left(\frac{1}{n^2+1}+\frac{1}{n^2+2}+\ldots+\frac{1}{n^2+2n}\right)$,

3.31. $\lim\limits_{n\to\infty} \left(\frac{1}{n^2+1}+\frac{2}{n^2+2}+\ldots+\frac{n}{n^2+n}\right)$,

3.32. $\lim\limits_{n\to\infty} (-1)^n \frac{n+\sin n!}{n+\cos n!}$,

3.33. $\lim\limits_{n\to\infty} (-1)^n \frac{\operatorname{arctg}\ n}{\operatorname{arcctg}\ n!}$,

3.34. $\lim\limits_{n\to\infty} n\left(\sqrt[n]{a}-1\right),\ a=\text{const}>0$,

3.35. $\lim\limits_{n\to\infty} \left[(n+1)^k-n^k\right],\ k=\text{const}\in(0,1)$,

3.36. $\lim\limits_{n\to\infty} \sqrt[n]{a^{2n}+a^{4n}},\ a=\text{const}\in\mathbf{R}$.