Cracow, July 2, 2024

FAM (WMS), Mathematics, group 1
RESIT TEST NO 1
INTRODUCTION TO PARTIAL DIFFERENIAL EQUATIONS

1. Find a solution of the equation

$$(x-y)u_x + (y-x-u)u_y = u$$

that passes through the curve:

$$x = s, \quad y = 3s, \quad u = 1.$$

2. Reduce the equation to canonical form

$$u_{xx} + u_{xy} + u_{yy} = 0.$$

3. Solve the initial-boundary problem

$$\begin{cases} u_{tt} = u_{xx}, & x \in (0,1),\ t > 0 \\ u(0,t) = 3, \quad u(1,t) = 0, & t > 0 \\ u(x,0) = 0, & x \in [0,1] \\ u_t(x,0) = x, & x \in [0,1] \end{cases}.$$

4. Solve the boundary problem

$$\begin{cases} u_{xx} + u_{yy} = y \sin x + \sin 2x, & (x,y) \in (0,2\pi) \times (0,1) \\ u(0,y) = u(2\pi,y) = 0, & y \in [0,1] \\ u(x,0) = 0,\ u(x,1) = 0, & x \in [0,2\pi] \end{cases}.$$

Check whether appropriate series are uniformly convergent.

5. A function is given

$$\varphi(x) = \begin{cases} 4 - x^2, & |x| \leqslant 2 \\ 0, & |x| > 2 \end{cases}.$$

Solve the initial problem (leave only non-elementary proper integrals $\int_{\alpha(t,x)}^{\beta(t,x)} e^{-y^2}\,dy$)

$$\begin{cases} u_t = u_{xx}, & x \in \mathbb{R},\ t > 0 \\ u(x,0) = \varphi(x), & x \in \mathbb{R} \end{cases}.$$

Draw the graph of the initial function φ and check the appropriate assumptions. Find an estimate of the solution u.

GOOG LUCK!!!:)