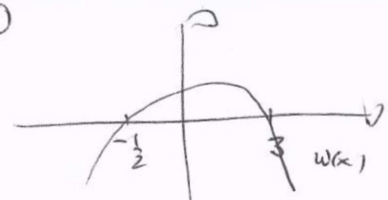
(1) $f(x) = x - \frac{3}{7} \ln \frac{2x+1}{3-x}$ $\frac{2x+1}{3-x} > 0$

$\mathcal{D} = \left(-\frac{1}{2}, 3\right)$ 0.5



$f'(x) = 1 - \frac{3}{7} \cdot \frac{\cancel{3-x}}{2x+1} \cdot \frac{2(3-x)+(2x+1)}{(3-x)^{\cancel{2}}} =$ $\underbrace{(2x+1)(3-x)}_{w(x)} > 0$

$= 1 - \frac{3}{\cancel{7}} \frac{\cancel{7}}{(2x+1)(3-x)} = \frac{(2x+1)(3-x) - 3}{(2x+1)(3-x)} = \frac{-2x^2+5x}{(2x+1)(3-x)} =$

$= \frac{-x(2x-5)}{(2x+1)(3-x)}$

$w(x) = -x(2x-5)$

$x_1 = 0, \ x_2 = \frac{5}{2} \in \mathcal{D}$ 2

$x \in \left(-\frac{1}{2}, 0\right)$, $f'(x) < 0$,	$f \downarrow$	$x_1 = 0$ мин. точ.	2.5
$x \in \left(0, \frac{5}{2}\right)$, $f'(x) > 0$,	$f \uparrow$		
$x \in \left(\frac{5}{2}, 3\right)$, $f'(x) < 0$,	$f \downarrow$	$x_2 = \frac{5}{2}$ макс. точ.	

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(2) $f(x) = \frac{x^3}{x+1}$ $\mathcal{D} = \mathbb{R} \setminus \{-1\}$ 0.5

$f'(x) = \frac{3x^2(x+1) - x^3}{(x+1)^2} = \frac{2x^3+3x^2}{(x+1)^2}$

$f''(x) = \frac{(6x^2+6x)(x+1)^{\cancel{2}} - (2x^3+3x^2)\, 2\cancel{(x+1)}}{(x+1)^{\cancel{4}\,3}} =$

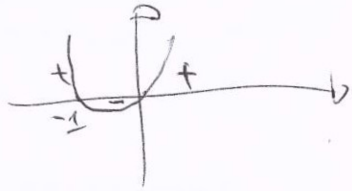
$= \frac{6x^3+6x^2+\cancel{6x^2}+6x-4x^3-\cancel{6x^2}}{(x+1)^3} = \frac{2x^3+6x^2+6x}{(x+1)^3} =$

$= \frac{2x(x^2+3x+3)}{(x+1)^3}$

$x^2+3x+3 = 0$
$\Delta = 9 - 12 = -3 < 0$

$x_0 = 0 \in \mathcal{D}$ 2

$x^2+3x+3 > 0 \ \forall x$



$w(x) = x(x+1)$

$x \in (-\infty, -1)$, $f''(x) > 0$,	$f \cup$		2.5
$x \in (-1, 0)$, $f''(x) < 0$,	$f \cap$	$x_0 = 0$ ПР	
$x \in (0, \infty)$, $f''(x) > 0$,	$f \cup$		

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(4) $f(x) = \frac{x^3+5}{3x^2-3x} = \frac{x^3+5}{3x(x-1)}$ $\mathcal{D} = \mathbb{R} \setminus \{0, 1\}$ 0.5

• $\lim\limits_{x\to\pm\infty} \frac{x^3+5}{3x^2-3x} = \lim \frac{x^{\cancel{3}}\left(1+\frac{5}{x^3}\right)}{x^{\cancel{2}}\left(3-\frac{3}{x}\right)} = \pm\infty$ $\boxed{\text{Нема хоризонтална}}$

•• $a = \lim\limits_{x\to\pm\infty} \frac{x^3+5}{x(3x^2-3x)} = \frac{1}{3}$, $b = \lim\limits_{x\to\pm\infty} \left(\frac{x^3+5}{3x^2-3x} - \frac{1}{3}x\right) = \lim \frac{\cancel{x^3}+5-\cancel{x^3}+x^2}{3(x^2-x)} = \frac{1}{3}$

$\boxed{y = \frac{1}{3}x + \frac{1}{3} \text{ коса асимпт.}}$ 2.5

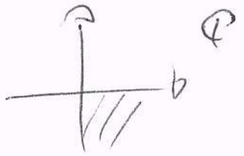
••• $\lim\limits_{x\to 0^\pm} \frac{x^3+5}{3x(x-1)} \overset{\frac{5}{0^\mp}}{=} \mp\infty$ $\boxed{x = 0 \text{ верт. ас.}}$

$\lim\limits_{x\to 1^\pm} \frac{x^3+5}{3x(x-1)} \overset{\frac{6}{0^\pm}}{=} \pm\infty$ $\boxed{x = 1 \text{ верт. ас.}}$ 2

(4) $2^8 z^5 + (\sqrt{3} - i)^{12} z^2 = 0$

$\boxed{z=0}$, $2^8 z^3 + (\underbrace{\sqrt{3} - i}_{z_1})^{12} = 0$

0.5



$|z_1| = 2$

$\varphi_1 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$(\sqrt{3} - i)^{12} = 2^{12}(\cos 22\pi + i \sin 22\pi) = 2^{12}$

$2^8 z^3 + 2^{12} = 0$

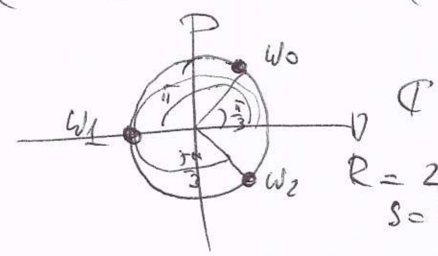
$z^3 = -8$ 1.5

$z \in \sqrt[3]{-8} = \left\{ w_k = 2\left(\cos \frac{\pi + 2k\pi}{3} + i \sin \frac{\pi + 2k\pi}{3}\right) : k = 0, 1, 2 \right\}$ ~~1.5~~

$w_0 = 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right) = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) = \boxed{1 + \sqrt{3} i}$ 2

$w_1 = 2(\cos \pi + i \sin \pi) = \boxed{-2}$

$w_2 = 2\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right) = 2\left(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3}\right) = 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) = \boxed{1 - \sqrt{3} i}$



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$R = 2$

$S = (0, 0)$