(1) $f(x)=\frac{1}{2}x^2 - x^2 \ln\frac{1}{x}$ $\qquad \frac{1}{x}>0,\ \underline{\mathcal{D}=(0,\infty)}$ (0.5)

$f'(x)= x - 2x\ln\frac{1}{x} - \cancel{x^2}\cdot x\cdot\left(-\frac{1}{\cancel{x^2}}\right) = x - 2x\ln\frac{1}{x} + x = 2x\left(1-\ln\frac{1}{x}\right)$

$2x\left(1-\ln\frac{1}{x}\right)=0$

$x_1 = 0 \notin \mathcal{D},\quad 1-\ln\frac{1}{x}=0$

$\ln\frac{1}{x}=1$

$\frac{1}{x}=e,\quad \underline{x_2=\frac{1}{e}\in\mathcal{D}}$ (2)

$x\in(0,\frac{1}{e}),\ f'(x)<0,\ f\downarrow$

$x\in(\frac{1}{e},\infty),\ f'(x)>0,\ f\uparrow$ $\quad$ $x_0=\frac{1}{e}$ min. tač. (2,5)

(*) $1-\ln\frac{1}{x}>0$

$\ln\frac{1}{x}<1$

$\ln\frac{1}{x}<\ln e$

$\frac{1}{x}<e$

$x>\frac{1}{e}$

---

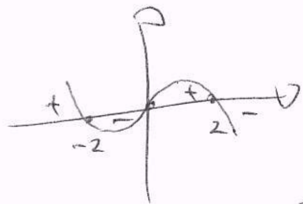
(2) $f(x)=\frac{x^3}{4-x^2}$ $\qquad \underline{\mathcal{D}=\mathbb{R}\setminus\{\pm 2\}}$ (0.5)

$f'(x)=\frac{3x^2(4-x^2)-x^3(-2x)}{(4-x^2)^2}=\frac{12x^2-x^4}{(4-x^2)^2}$

$f''(x)=\frac{(24x-4x^3)(4-x^2)^{\cancel{2}}-(12x^2-x^4)2\cancel{(4-x^2)}(-2x)}{(4-x^2)^{\cancel{4}\,3}}=$

$=\frac{96x-24x^3-16x^3+\cancel{4x^5}+48x^3-\cancel{4x^5}}{(4-x^2)^3}=\frac{96x+8x^3}{(4-x^2)^3}=$

$=\frac{x(96+8x^2)}{(4-x^2)^3}$



$w(x)=x(4-x^2)$

$x_0=0\in\mathcal{D}$ (2)

$x\in(-\infty,-2),\ f''(x)>0,\ f\cup$

$x\in(-2,0),\ f''(x)<0,\ f\cap$ $\quad$ $x_0=0$ PP

$x\in(0,2),\ f''(x)>0,\ f\cup$

$x\in(2,\infty),\ f''(x)<0,\ f\cap$ (2,5)

---

(3) $f(x)=\frac{x^3}{x^2-x-2}=\frac{x^3}{(x-2)(x+1)}$ $\qquad x^2-x-2=0$, $\ \Delta=1+8=9$, $\ x=\frac{1\pm3}{2}$ $\qquad \mathcal{D}=\mathbb{R}\setminus\{2,-1\}$ (0.5)

• $\lim\limits_{x\to -1^\pm}\frac{x^3}{(x-2)(x+1)}\overset{\frac{-1}{0^\mp}}{=}\pm\infty$

$\lim\limits_{x\to 2^\pm}\frac{x^3}{(x-2)(x+1)}\overset{\frac{8}{0^\pm}}{=}\pm\infty$

$\boxed{x=-1,\ x=2 - \text{vert. as.}}$ (2)

•• $\lim\limits_{x\to\pm\infty}\frac{x^3}{x^2-x-2}=\pm\infty$ $\quad$ $\boxed{\text{Nema hor. asimptote}}$

••• $a=\lim\limits_{x\to\pm\infty}\frac{x^2}{x^2-x-2}=\underline{1}$

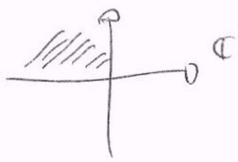
$b=\lim\limits_{x\to\pm\infty}\left(\frac{x^3}{x^2-x-2}-x\right)=\lim\limits_{x\to\pm\infty}\frac{\cancel{x^3}-\cancel{x^3}+x^2+2x}{x^2-x-2}=\underline{1}$

$\boxed{y=x+1 \text{ kosa asimptota}}$ (2,5)

(4) $(-\sqrt{3}+i)^{18} z^2 + 2^{15} z^5 = 0$

$\boxed{z=0}$ (0.5) $\qquad \underbrace{(-\sqrt{3}+i)}_{z_1}{}^{18} + 2^{15} z^3 = 0$

$|z_1| = 2$




$\varphi_1 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$(-\sqrt{3}+i)^{18} = 2^{18}(\cos 15\pi + i \sin 15\pi) = -2^{18}$

$-2^{18} + 2^{15} z^3 = 0$

$z^3 = 8$ (1.5)

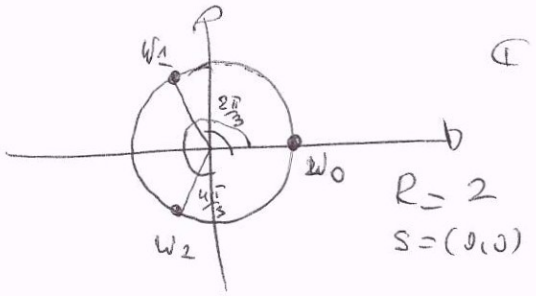
$z \in \sqrt[3]{8} = \left\{ w_k = 2\left(\cos \frac{2k\pi}{3} + i \sin \frac{2k\pi}{3}\right) : k = 0, 1, 2 \right\}$

$w_0 = 2(\cos 0 + i \sin 0) = \boxed{2}$

$w_1 = 2\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right) = 2\left(\cos\left(\pi - \frac{\pi}{3}\right) + i \sin\left(\pi - \frac{\pi}{3}\right)\right) = 2\left(-\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right) = 2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) =$
$= \boxed{-1 + \sqrt{3} i}$

$w_2 = 2\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right) = 2\left(\cos\left(\pi + \frac{\pi}{3}\right) + i \sin\left(\pi + \frac{\pi}{3}\right)\right) =$

$= 2\left(-\cos\frac{\pi}{3} - i \sin\frac{\pi}{3}\right) = 2\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) = \boxed{-1 - \sqrt{3} i}$ (2)




$R = 2$
$S = (0, 0)$ (1)