Computer-aided proofs in dynamics III

Blenders

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joint work with

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Plan of the presentation

- Toy example
- Cone conditions and covering relations
- Wall property and blender construction
- Computer assisted proof

- Greetings from Daniel Wilczak:

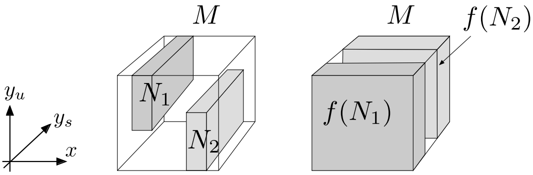
  `git clone https://github.com/CAPDGroup/CAPDexercises`

Toy example

$$f : \mathbb{R}^3 \to \mathbb{R}^3$$

diffeomorphism

- x – strong expanding
- y_u – weak expanding
- y_s – contracting



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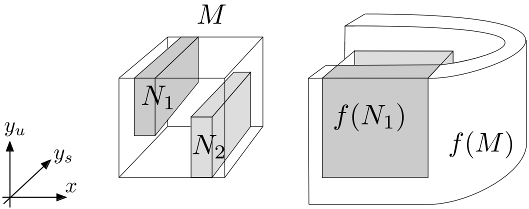

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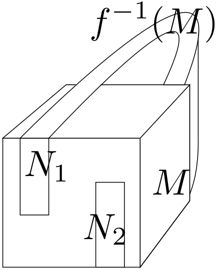
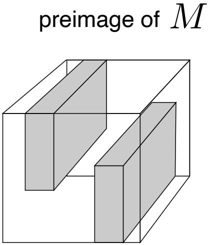
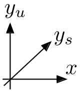




$$\mathcal{G}(U) := f^{-1}(U) \cap M$$



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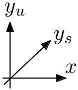
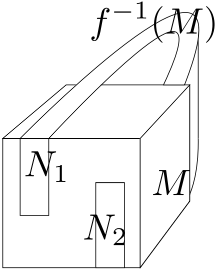
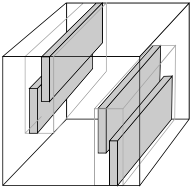




second preimage of M



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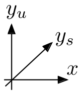

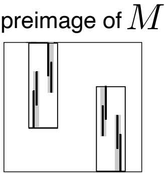

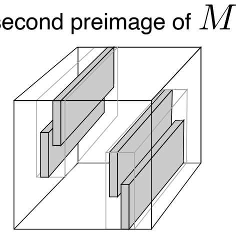

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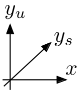

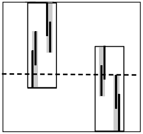

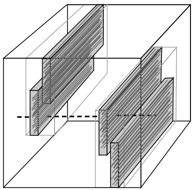

$$\mathcal{G}(U) := f^{-1}(U) \cap M$$

$$\Lambda = \mathrm{Inv}(f, M)$$

$$W^s(\Lambda) = \lim_{n\to\infty} \mathcal{G}^n(M)$$

$W^s(\Lambda)$ has one dimensional bundles, and yet behaves as a plane.

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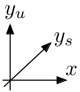
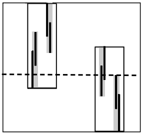
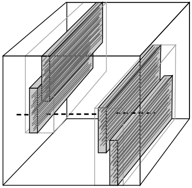


third preimage of M



in the limit



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[BCDW] Bonatti, Crovisier, Díaz, Wilkinson, What is. . . a blender? Notices Amer. Math. Soc. (2016)

Hyperbolic set

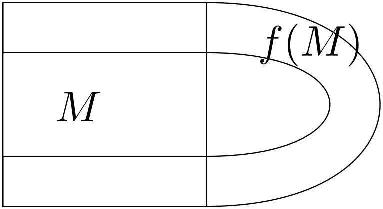
$$f : \mathbb{R}^n \to \mathbb{R}^n$$

Definition

We say that Λ is a hyperbolic set iff $T_z\Lambda = E^u_z \oplus E^s_z$, $u + s = n$

$$Df(z)E^u_z = E^u_{f(z)}$$
$$Df(z)E^s_z = E^s_{f(z)}$$
$$\|Df^n(z)v\| < c\lambda^n\|v\| \quad \text{for } v \in E^s_z$$
$$\|Df^{-n}(z)v\| < c\lambda^n\|v\| \quad \text{for } v \in E^u_z$$

where $z \in \Lambda$ and $\lambda \in (0, 1)$.



$$\Lambda = \mathrm{Inv}(f, M)$$

$$u = 1 \qquad s = 1$$

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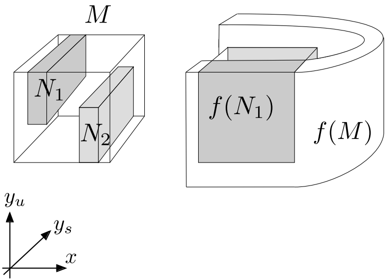


$$\Lambda = \mathrm{Inv}(f, M)$$

$$u = 2 \qquad s = 1$$

Hyperbolic set

$$f : \mathbb{R}^n \to \mathbb{R}^n$$

Theorem

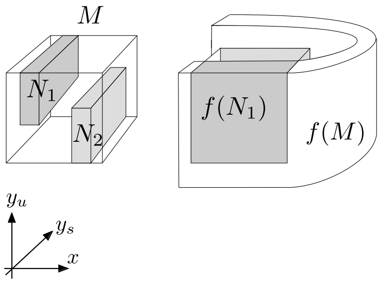
Let $\Lambda = \mathrm{Inv}(f, \bigcup N_i)$ *and let*

$$\mathcal{K} = \mathrm{diag}(Id_u, -Id_s).$$

If for $z \in f^{-1}(N_j) \cap N_i$

$$Df(z)^T \mathcal{K} Df(z) - \mathcal{K}$$

is positive definite, then Λ *is a hyperbolic set.*



[W] Wilczak, Uniformly hyperbolic attractor of the Smale-Williams type for a Poincaré map in the Kuznetsov system, SIADS (2010)

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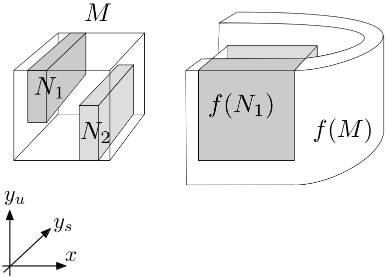
Blender

$$f : \mathbb{R}^n \to \mathbb{R}^n$$

Definition

We say that a hyperbolic set Λ is a k-blender if $s < k$ and there exists an open set W of $n-k$ dimensional surfaces such that

$$h \cap W^s_\Lambda \neq \emptyset \qquad \forall h \in W.$$



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Intuition:

- W^s_Λ is 's-dimensional', but 'behaves as' a k-dimensional surface.

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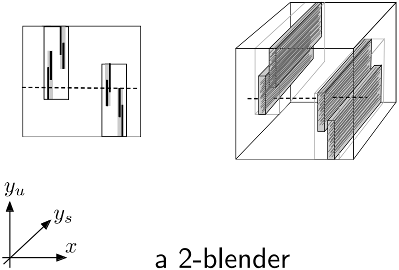

a 2-blender

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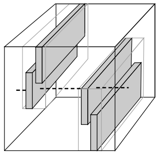
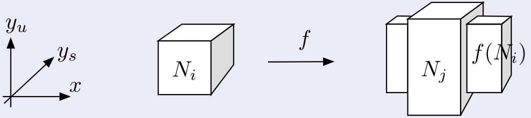
Covering relations



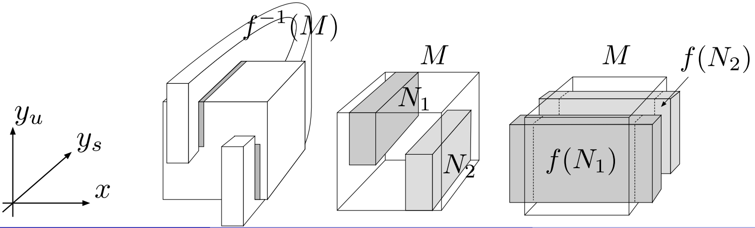
Definition

We say that $N_i \implies N_j$ iff



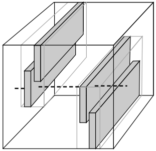
In the setting of the blender construction:

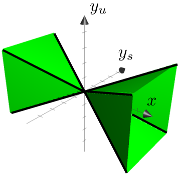
- 'Topological entry' coordinate $y = (y_s, y_u)$ includes y_u, which is expanding.



Cone conditions

$$C(z) = \{q : \|\pi_x(q - z)\| \geqslant \|\pi_y(q - z)\|\}$$



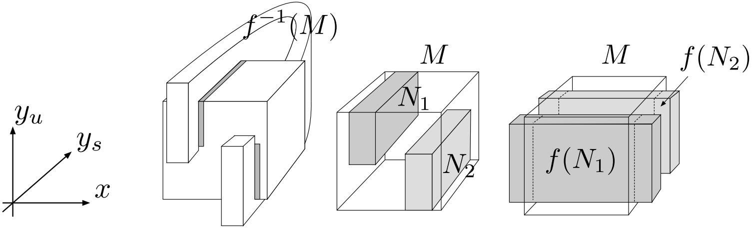


Definition

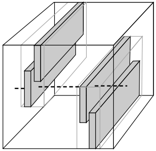
f satisfies cone conditions iff

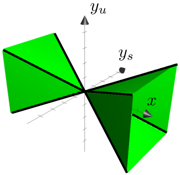
$$f(C(z)) \subset C(f(z))$$

Notation: $N \overset{cc}{\Longrightarrow} M$ means covering and cone conditions.



horizontal discs

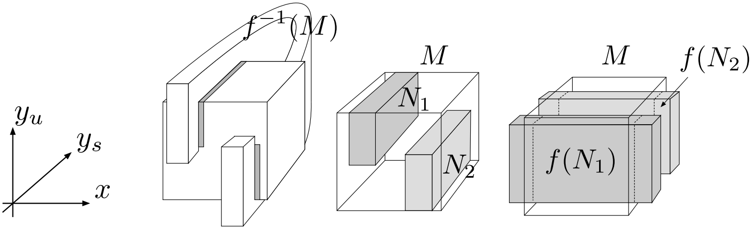




Definition

Let $N = B_x \times B_y$. We say that $h : B_x \to N$ is a horizontal disc in N iff

- $h(x) = (x, \pi_y h(x))$
- $h(B_x) \subset C(h(x))$



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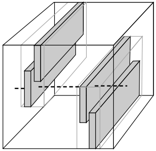

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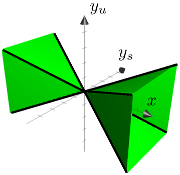

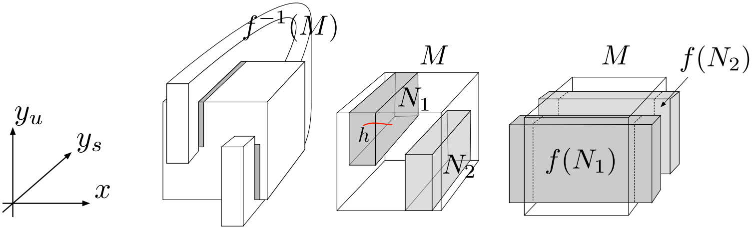

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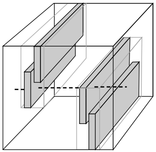

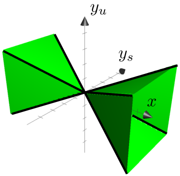

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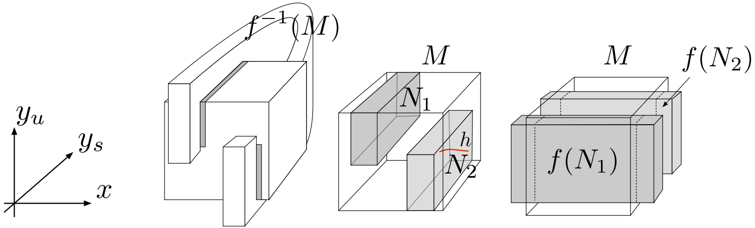

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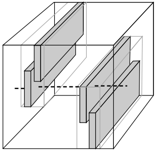

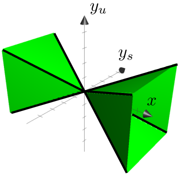

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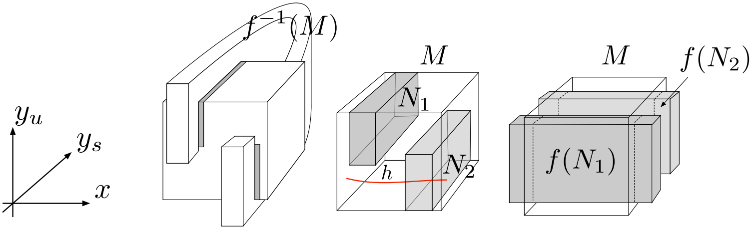
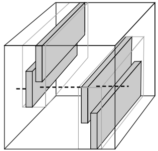


Main result

$M \in \{N_i\}_{i=1,\dots,m} \qquad \Lambda = \mathrm{Inv}(f, \bigcup N_i)$



Theorem

(A1) *For every horizontal disc $h \in M$ $\exists i, j$*
$h_i = h \cap N_i \qquad N_i \overset{cc}{\Longrightarrow} N_j$

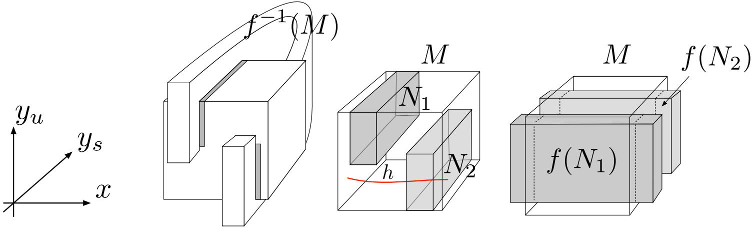
(A2) *$\forall i$ either $N_i = M$ or $\exists j$ s.t. $N_i \overset{cc}{\Longrightarrow} N_j$*

(A3) *Λ is hyperbolic*

If (A1–A3) then Λ is a k-blender

k is the dimension of the topological entry coordinate $y = (y_u, y_s)$ and $k > s$

below
$\{N_i\} = \{N_1, N_2, M\}$



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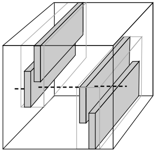

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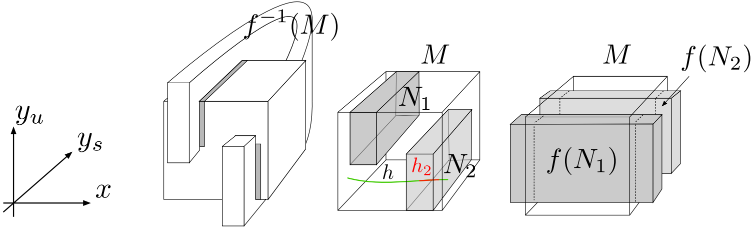

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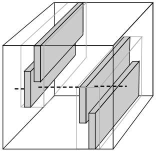

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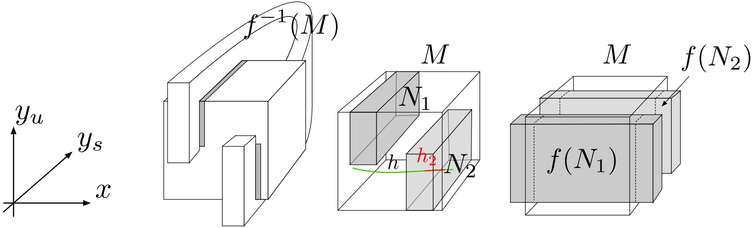

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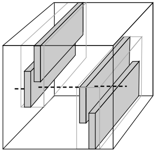

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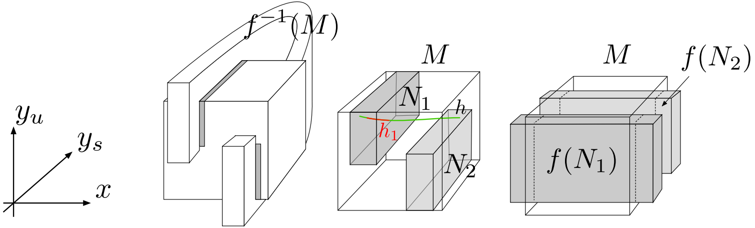

Main result - proof

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$\mathcal{G}(U) = f^{-1}(U) \cap \bigcup N_i$

Definition

$A \subset \bigcup N_i$ is a wall if for every $h_i \in N_i$

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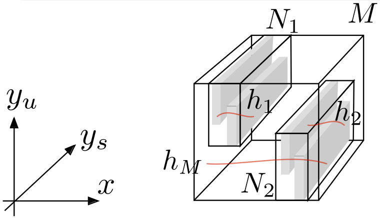
Lemma

If A is a wall and (A1–A2) then $\mathcal{G}(A)$ is a wall.



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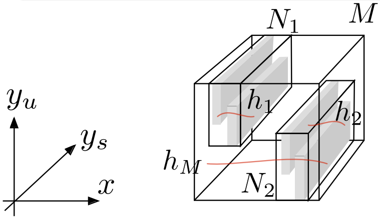

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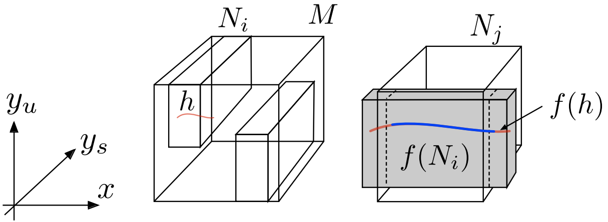

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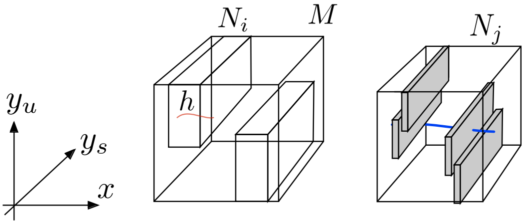

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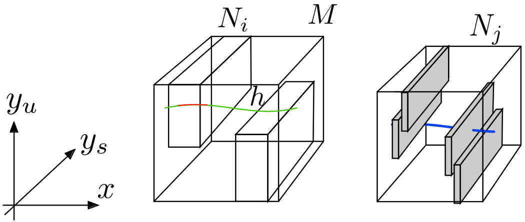

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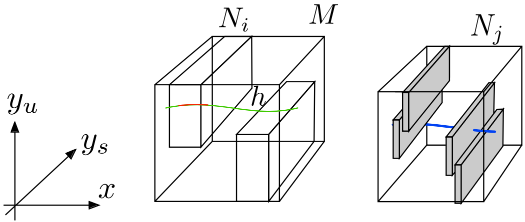

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If (A1–A3) then Λ *is a* k*-blender*

Proof. $A_0 = \bigcup N_i$ is a wall.

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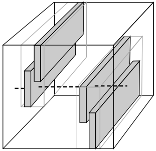

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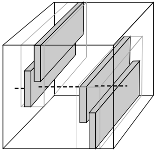

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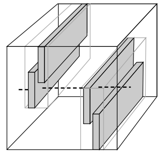

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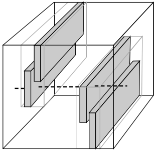

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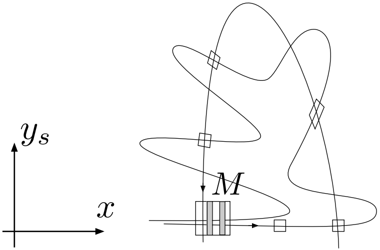
[BD] Bonatti, Díaz, Persistent nonhyperbolic transitive diffeomorphisms, Ann. of Math. (2), 143 (1996), 357–396,



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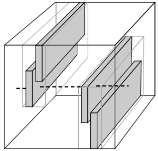

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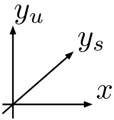

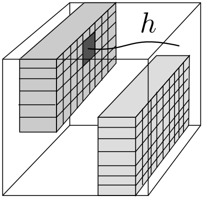

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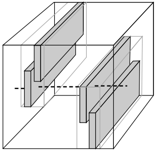

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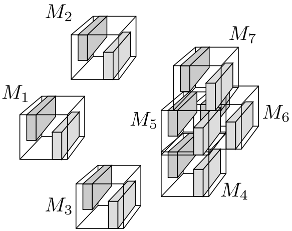

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Example

Hénon family

$$f(x,y,z) = \left(y, \mu + y^2 + \beta x, \xi z + y\right)$$

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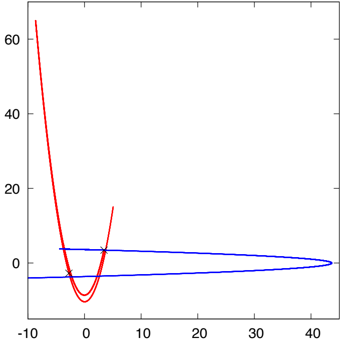
For every $\xi \in [1.01, 1.125]$ the system has a 2-blender.



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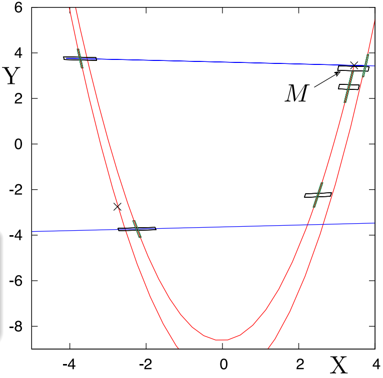

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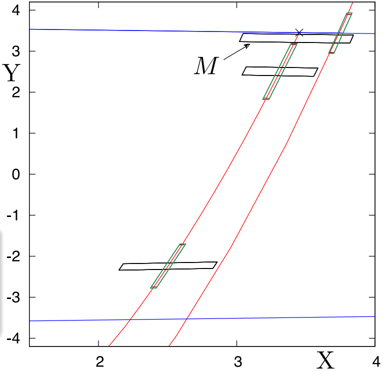

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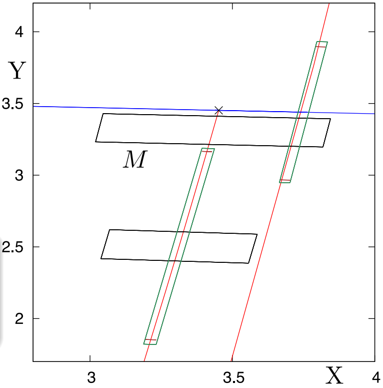

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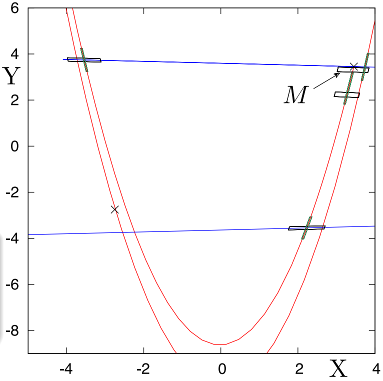

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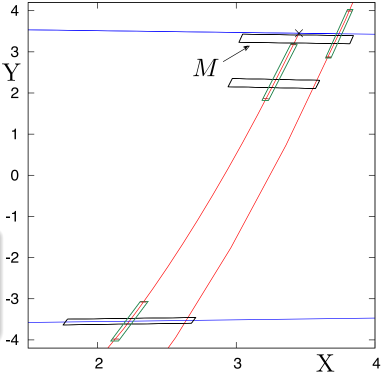

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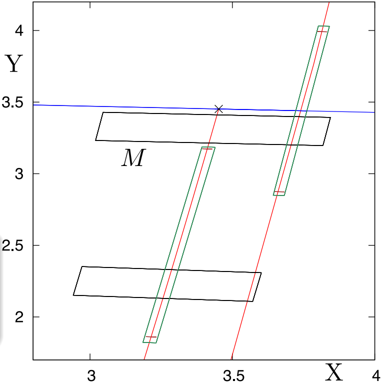

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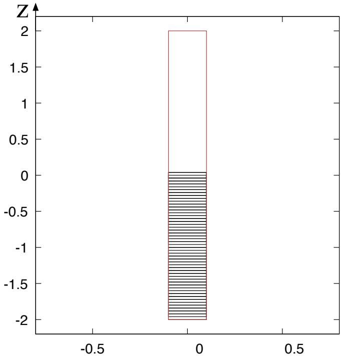

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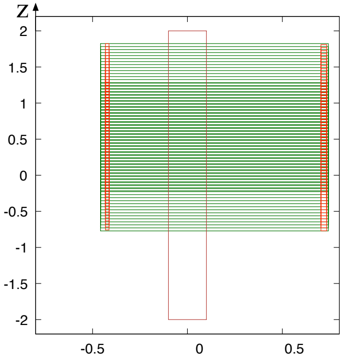

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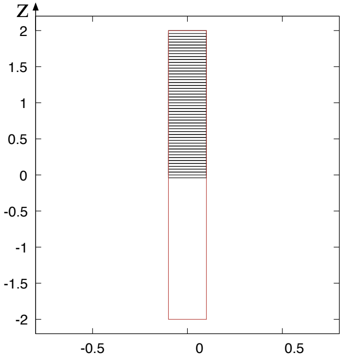

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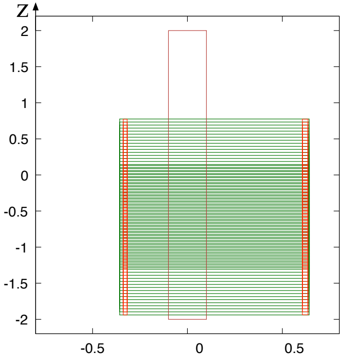

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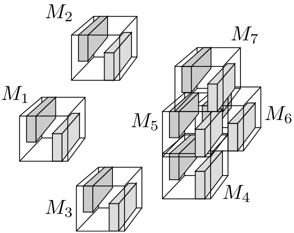
Closing remarks

- Blenders follow from covering relations and cone conditions
- Conditions (A1)–(A3) are verifiable using computer assisted proofs
- Flexible framework



Friday: Heterodimensional cycles

[CKOZ] MC, B. Krauskopf, H. Osinga, P. Zgliczyński, Characterising blenders via covering relations and cone conditions. arXiv