Vertex distinguishing colorings of graphs

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We shall consider

edge colorings of graphs and use these colorinings to distinguish the vertices of the graphs.

I. proper colorings

   (joint work with R. Kalinowski, M. Pilśniak and J. Przybyło form AGH University)

II. general colorings

   (joint work with O.Baudon and J.Bensmail from LaBRI and J. Przybyło form AGH University)

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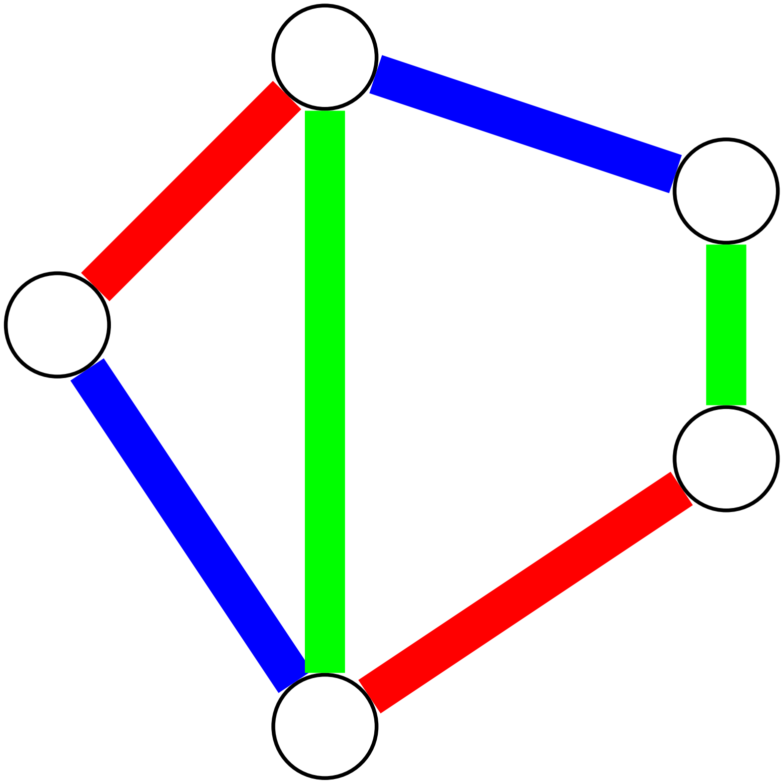
- A graph is Class 1 if $\chi'(G) = \Delta(G)$,
- A graph is Class 2 if $\chi'(G) = \Delta(G) + 1.$

Two colorings



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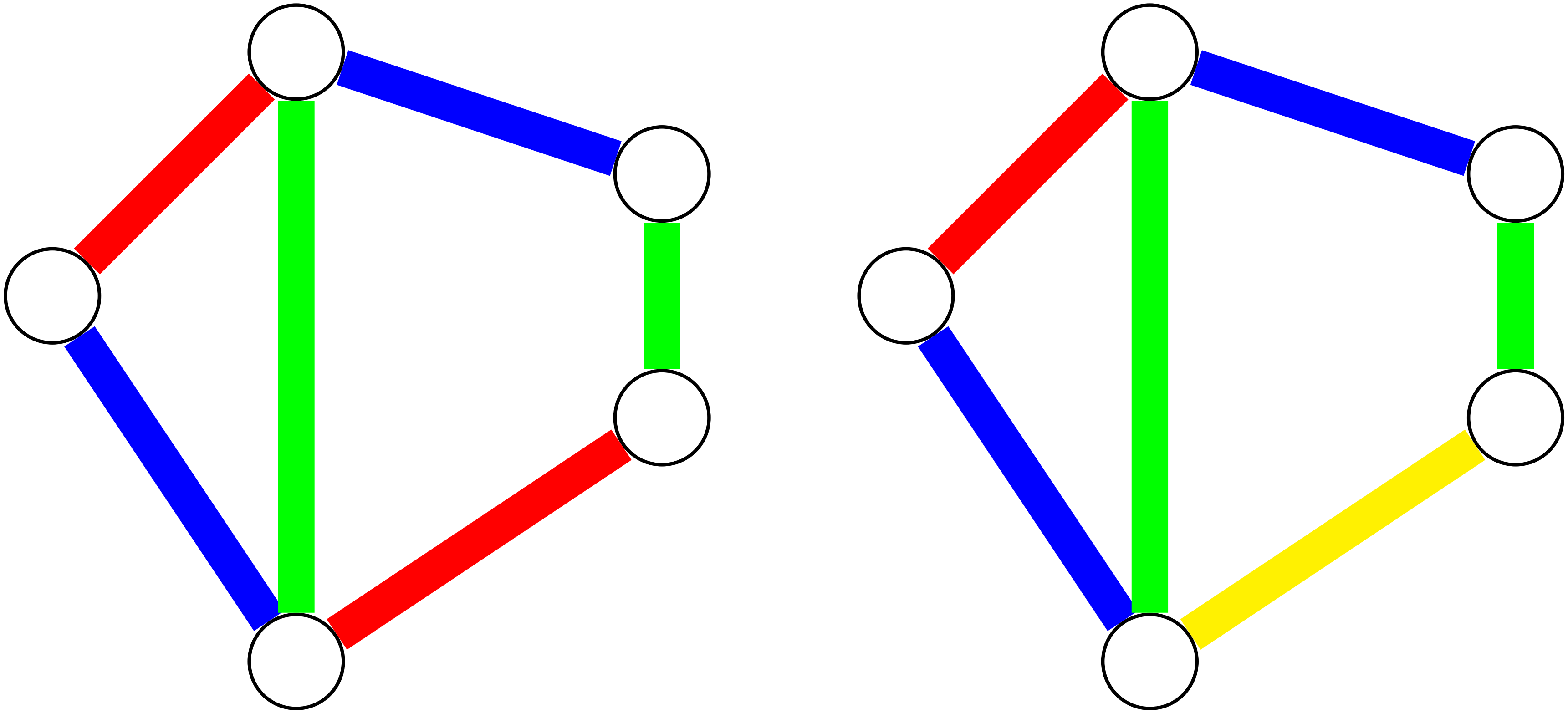

More definitions

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Coloring distinguishing (all) vertices

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- 1+2.
  Graph Theory Week, Banach Centre 1996 (Horňák + Favaron)

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- Remark. We assume that our graph has neither K_2 nor two K_1 as components.

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- Example (!): complete graphs K_{2p}.

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  Theorem:
  If $\delta(G) \geq \frac{n}{3}$, then vdi $(G) \leq \Delta(G) + 5$.

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- So, the number of all possible 2-element sets is $\binom{k}{2}$, if we use k colors,
- We have $\binom{k}{2} \geq n$
- $\mathrm{vdi} \geq \sqrt{2n}$.

By the way ...

- Conjecture 2.
  $\pi \leq \text{vdi}\,(G) \leq \pi + 1$
  where $\pi = \max_i \left\{ \min_k \left\{ k : \binom{k}{i} \geq n_i \right\} \right\}$
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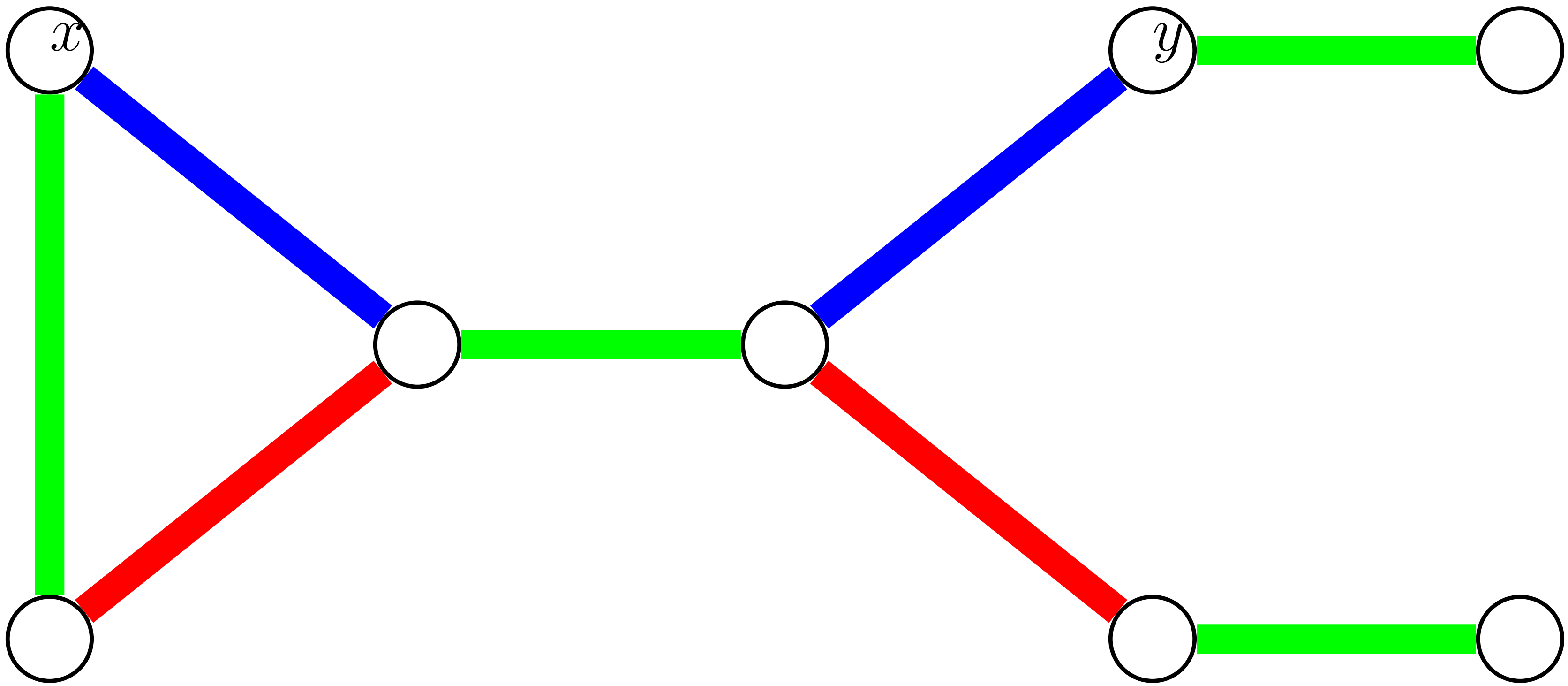
- Still open despite of many papers: mainly by P.N.Balister with:
  B.Bollobás, O.M.Riordan, R.H.Schelp, A.Kostoczka, Hao Li.

An example



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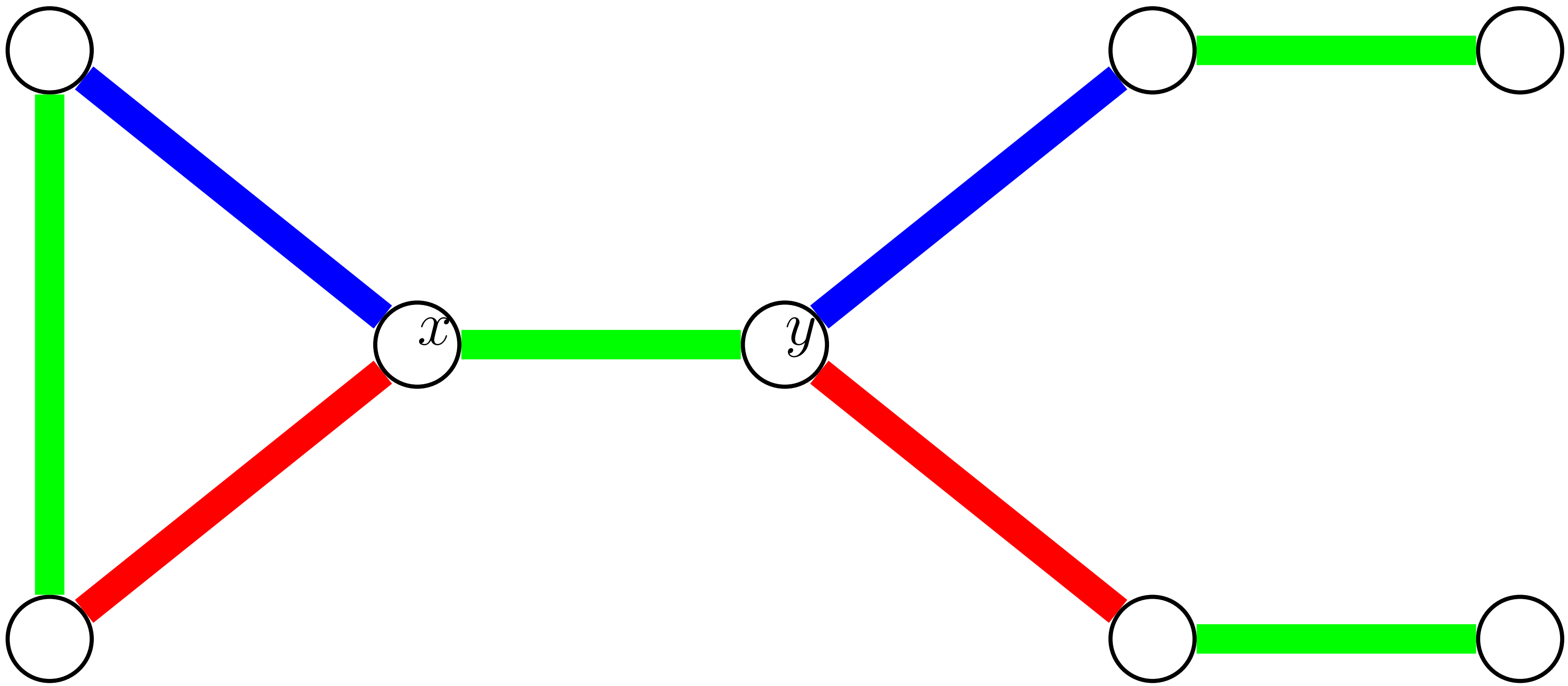
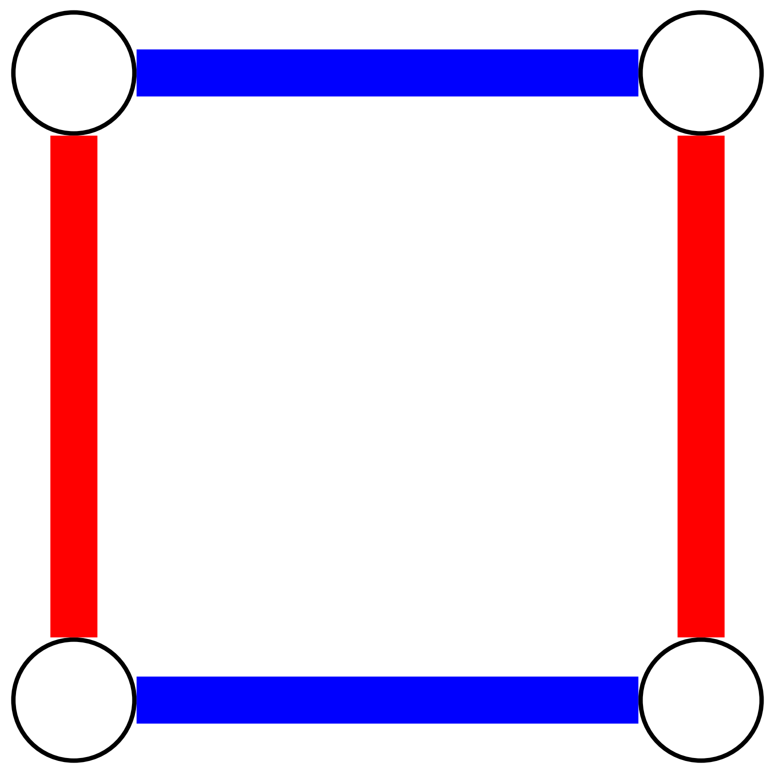


C_4



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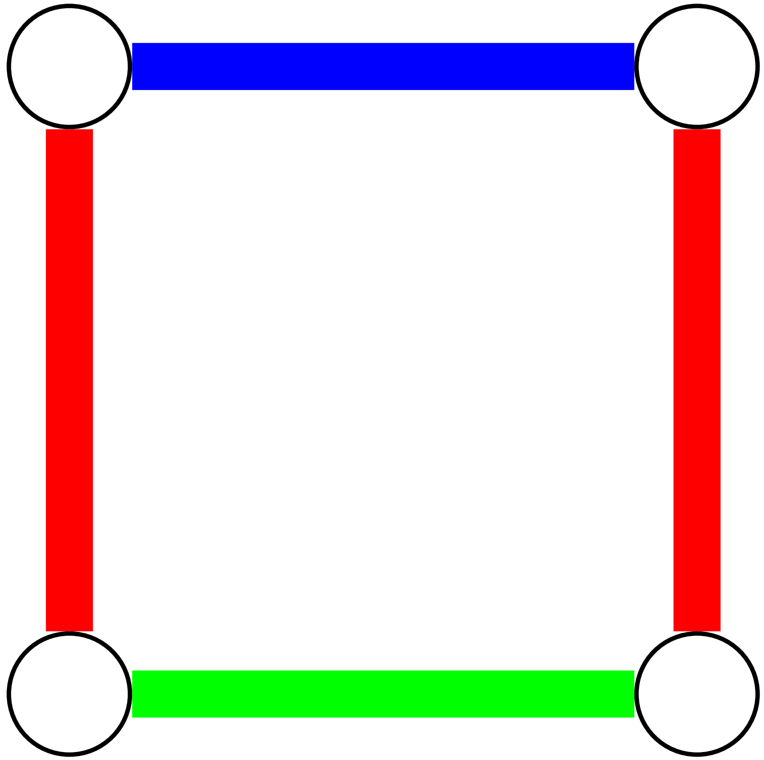

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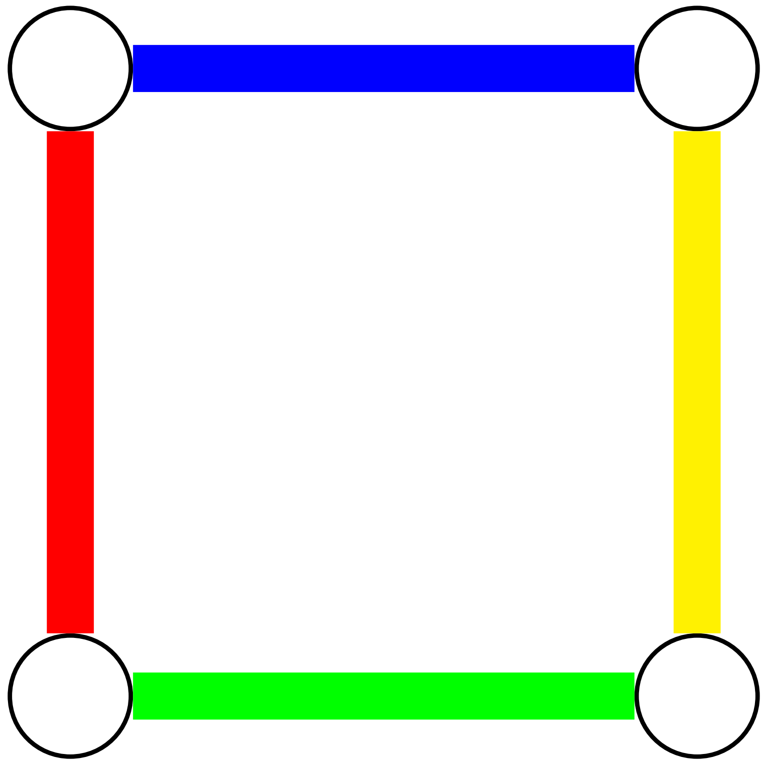

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- two vertices x and y are similar if $W(x) = W(y)$.
- $\mu(G)$ - the minimum number of colors in a proper edge-coloring of a graph G such that no two distinct vertices are similar.

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- R. Kalinowski, M. Pilśniak, J. Przybyło and M. Woźniak, *How to personalize the vertices of a graph?*, European Journal of Combinatorics, 40 (2014), 116–123.

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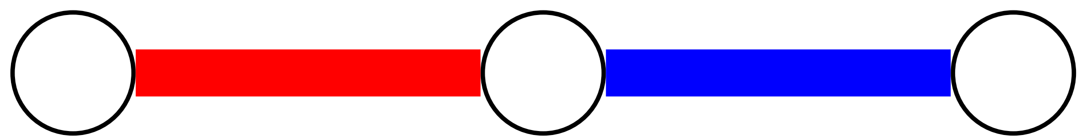
- (main theorem for graphs class I)

Proof



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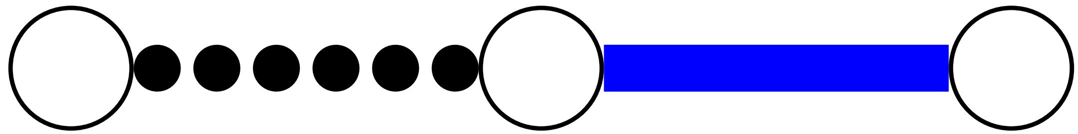

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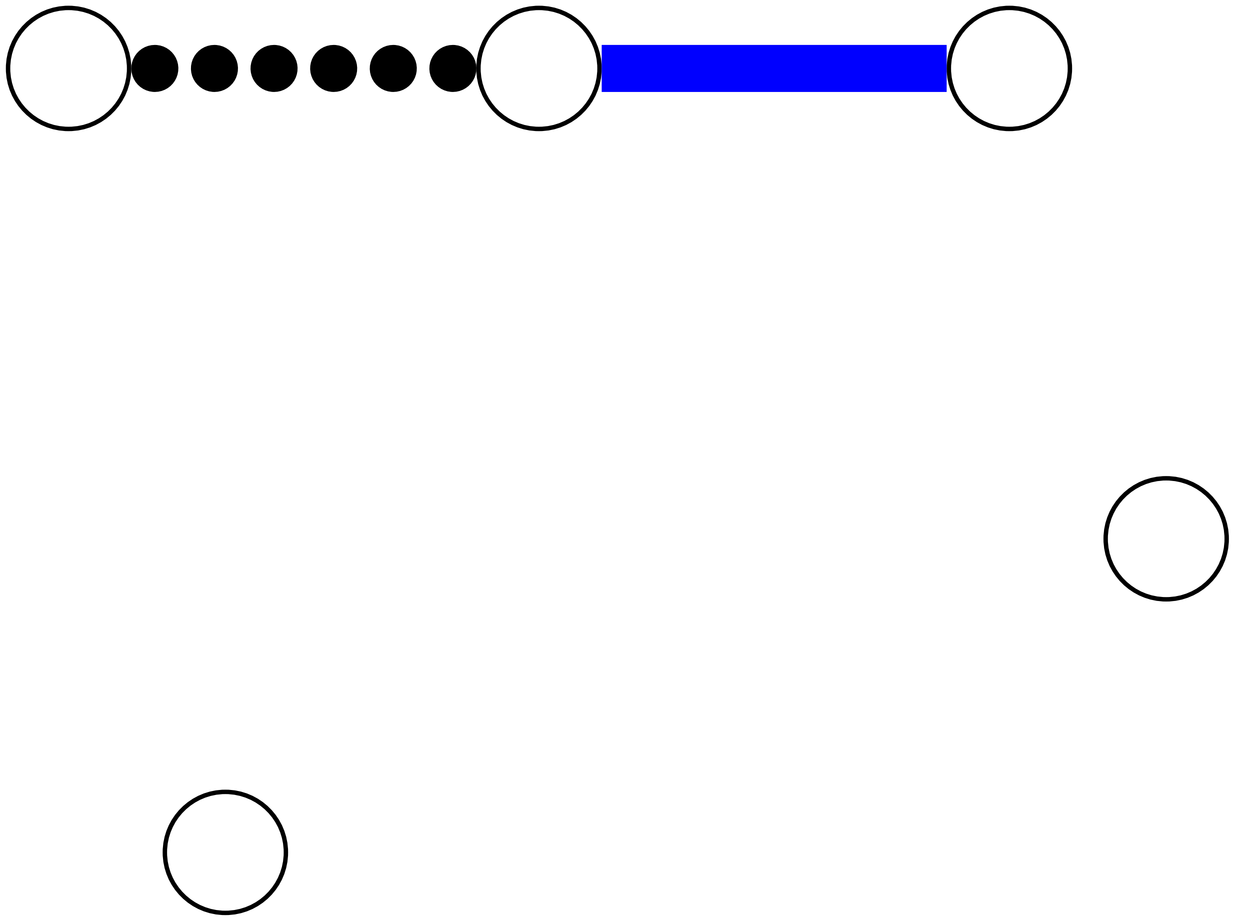

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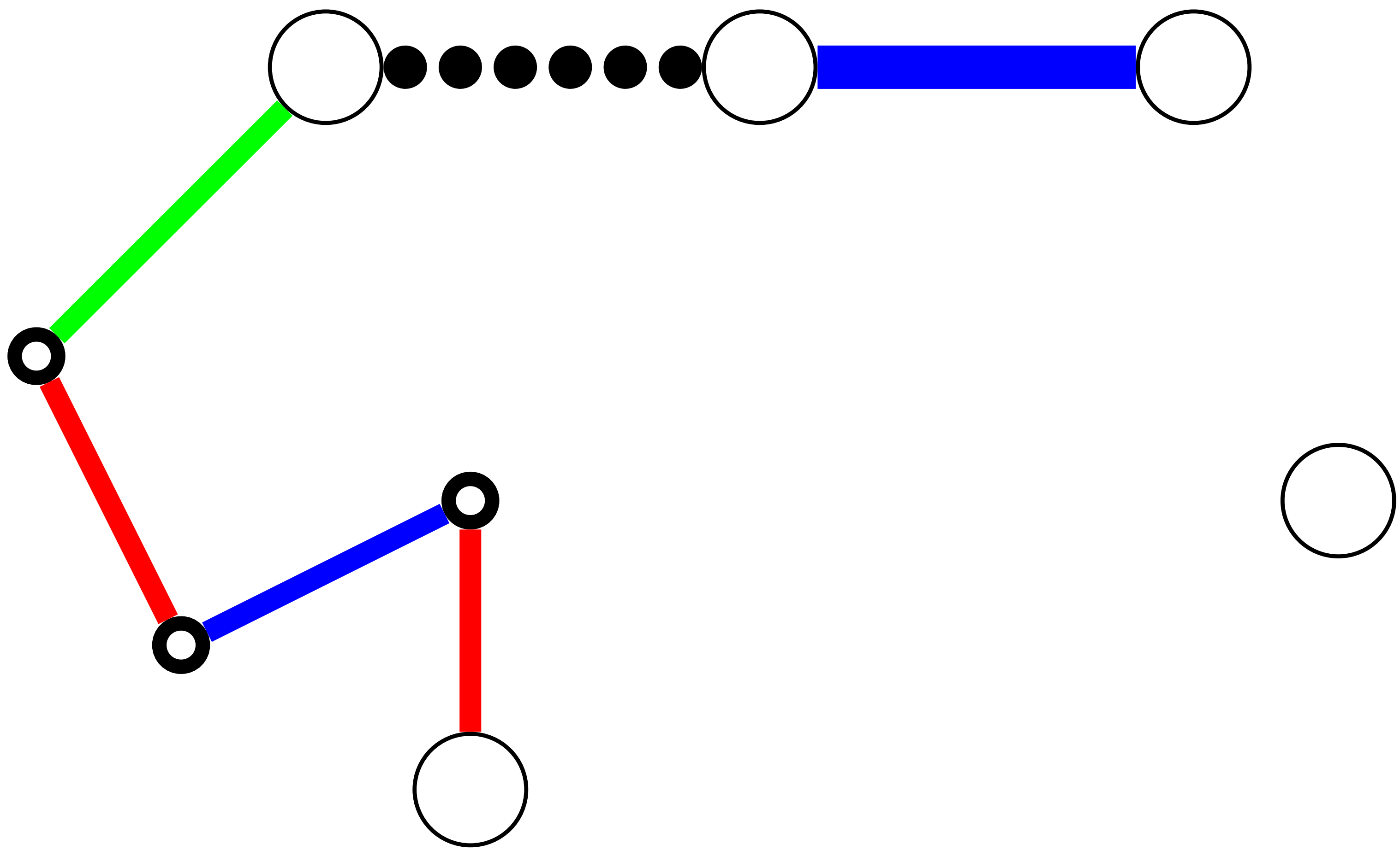

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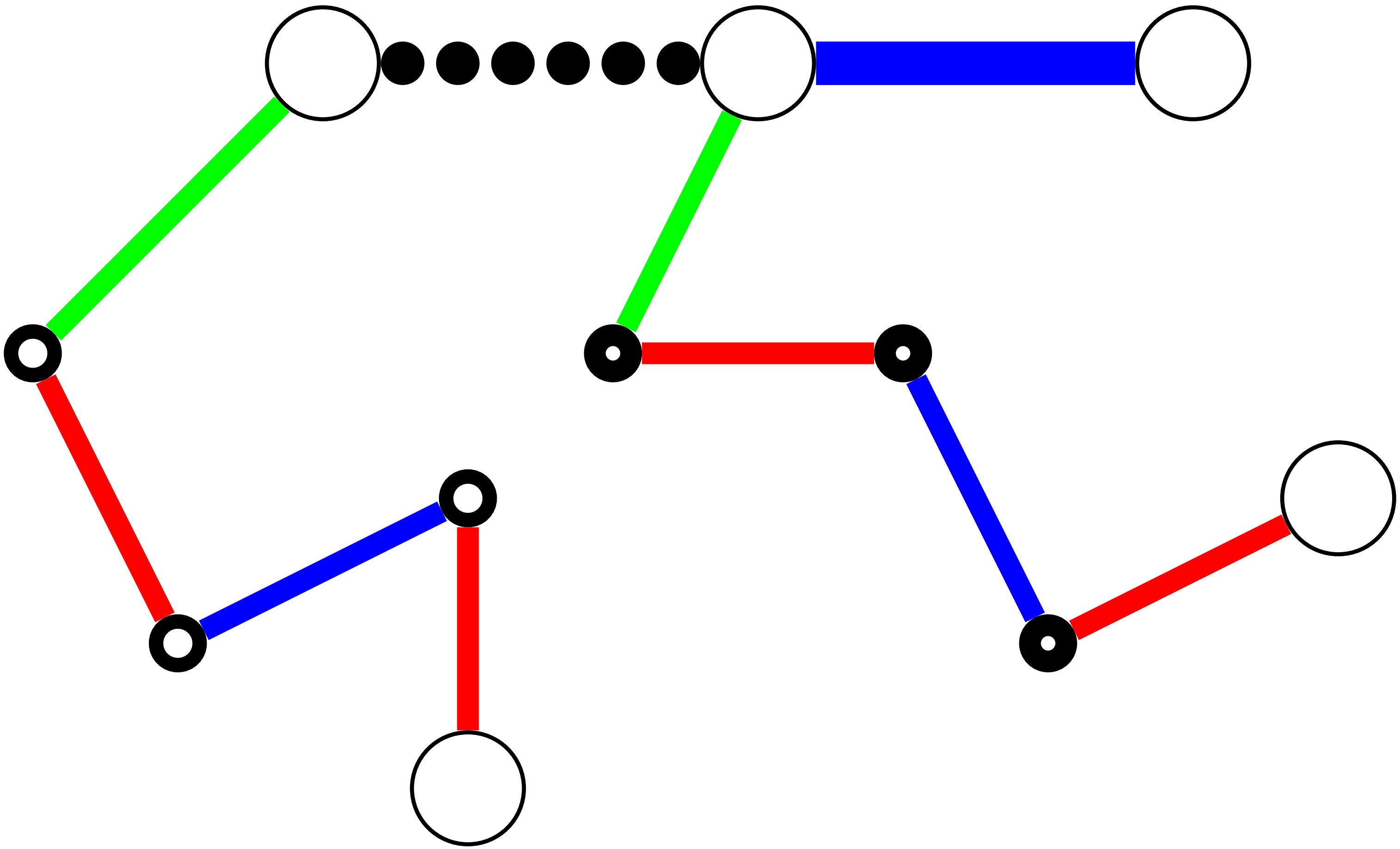
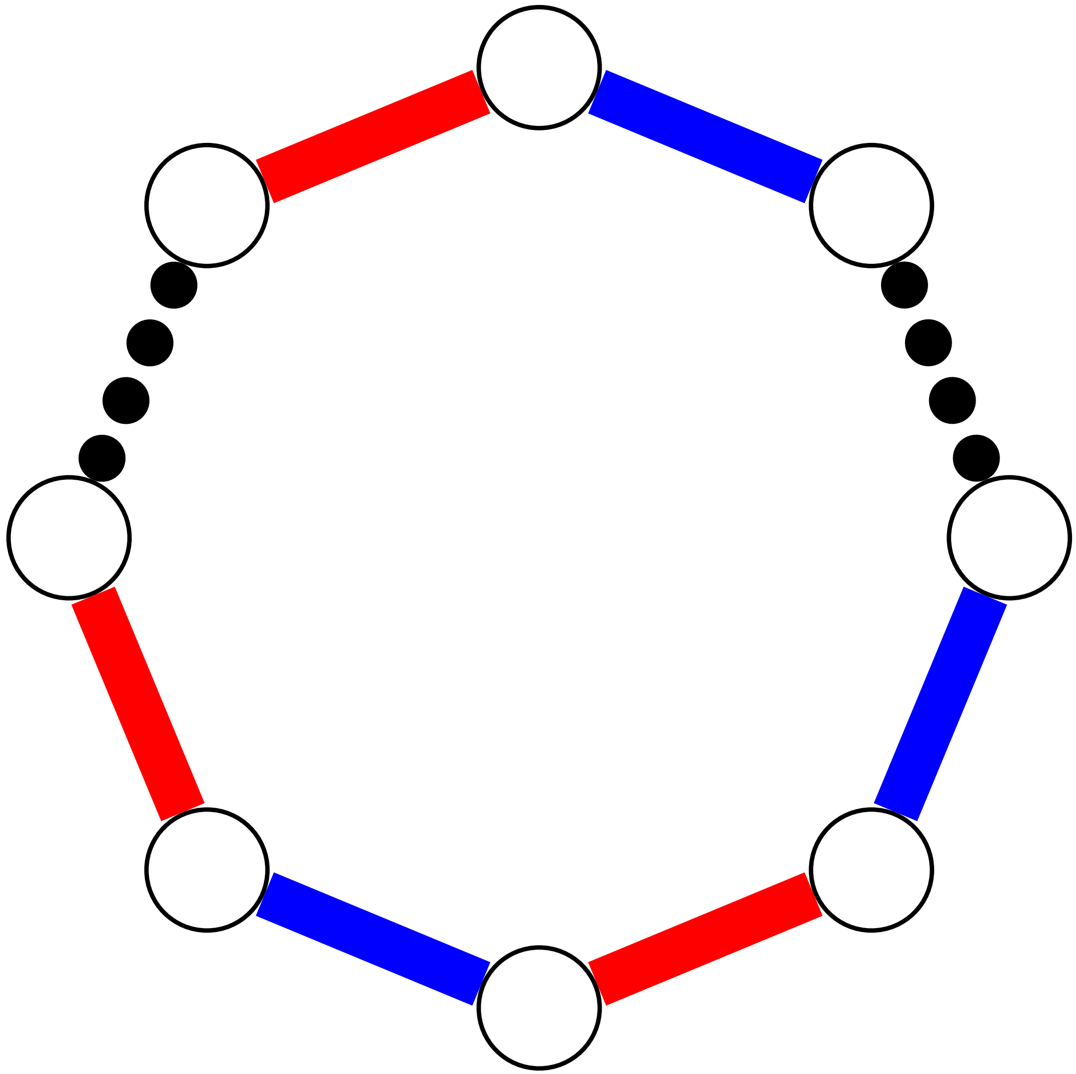
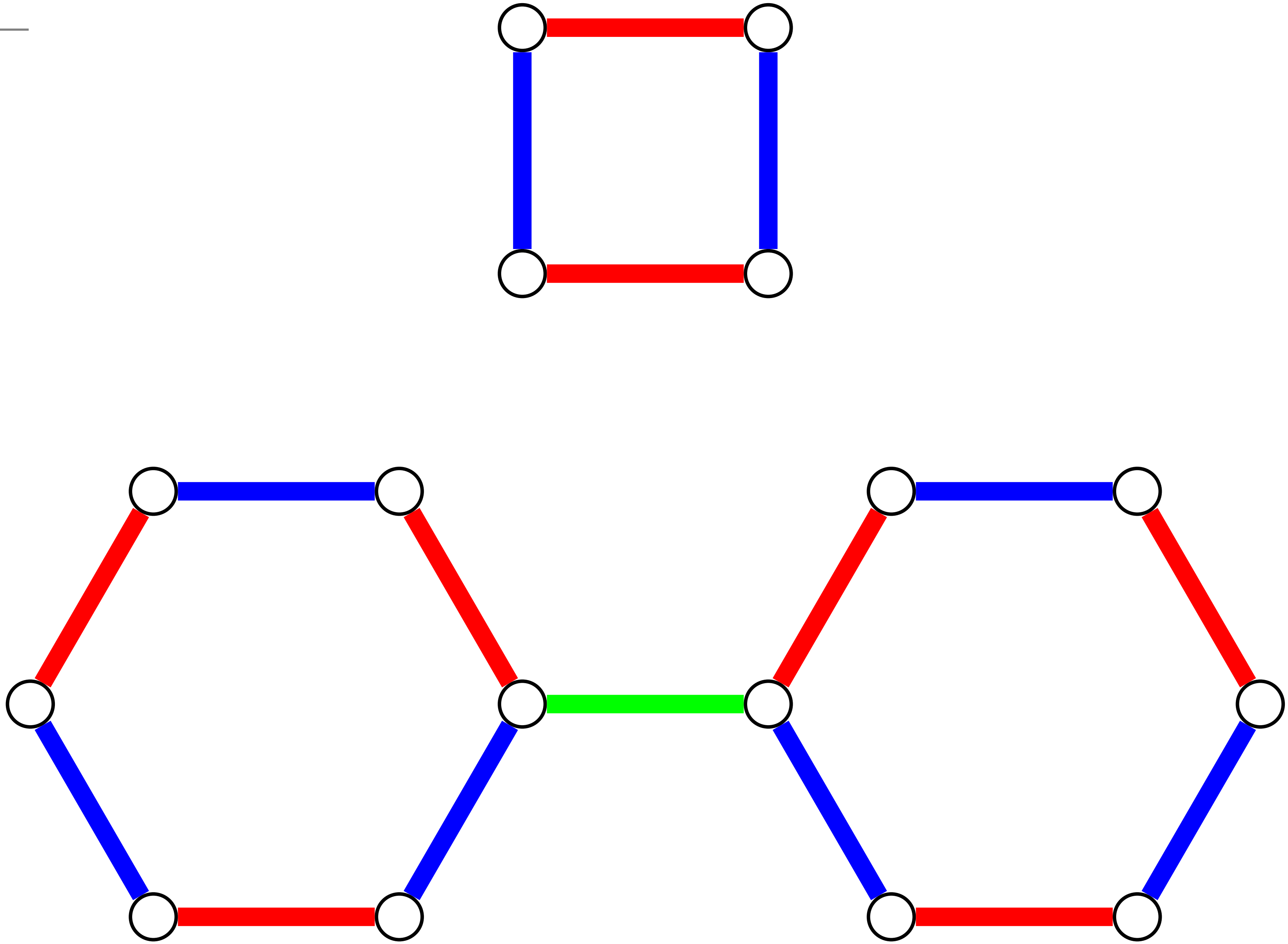


C_8 - a coloring



One of cases



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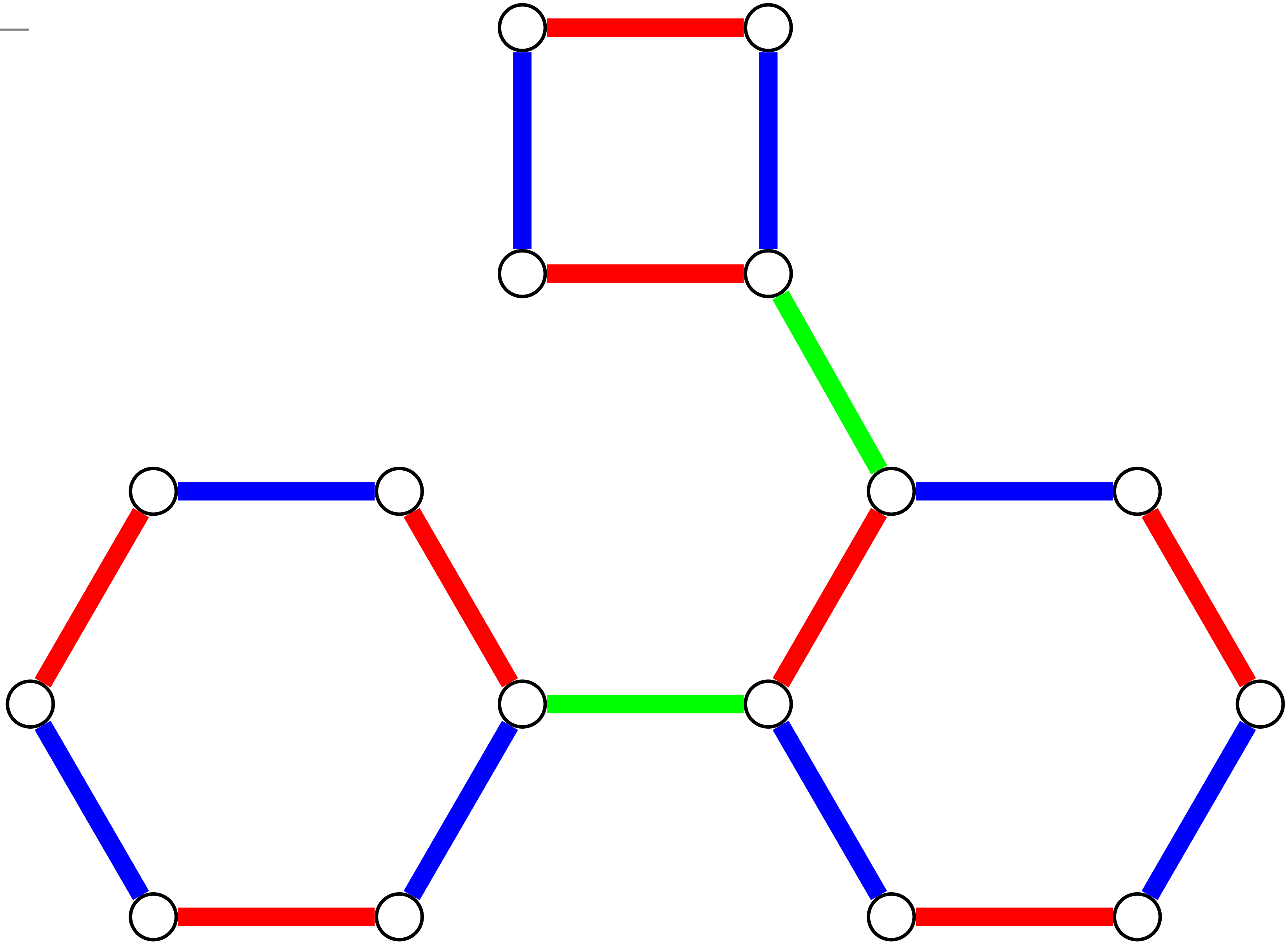

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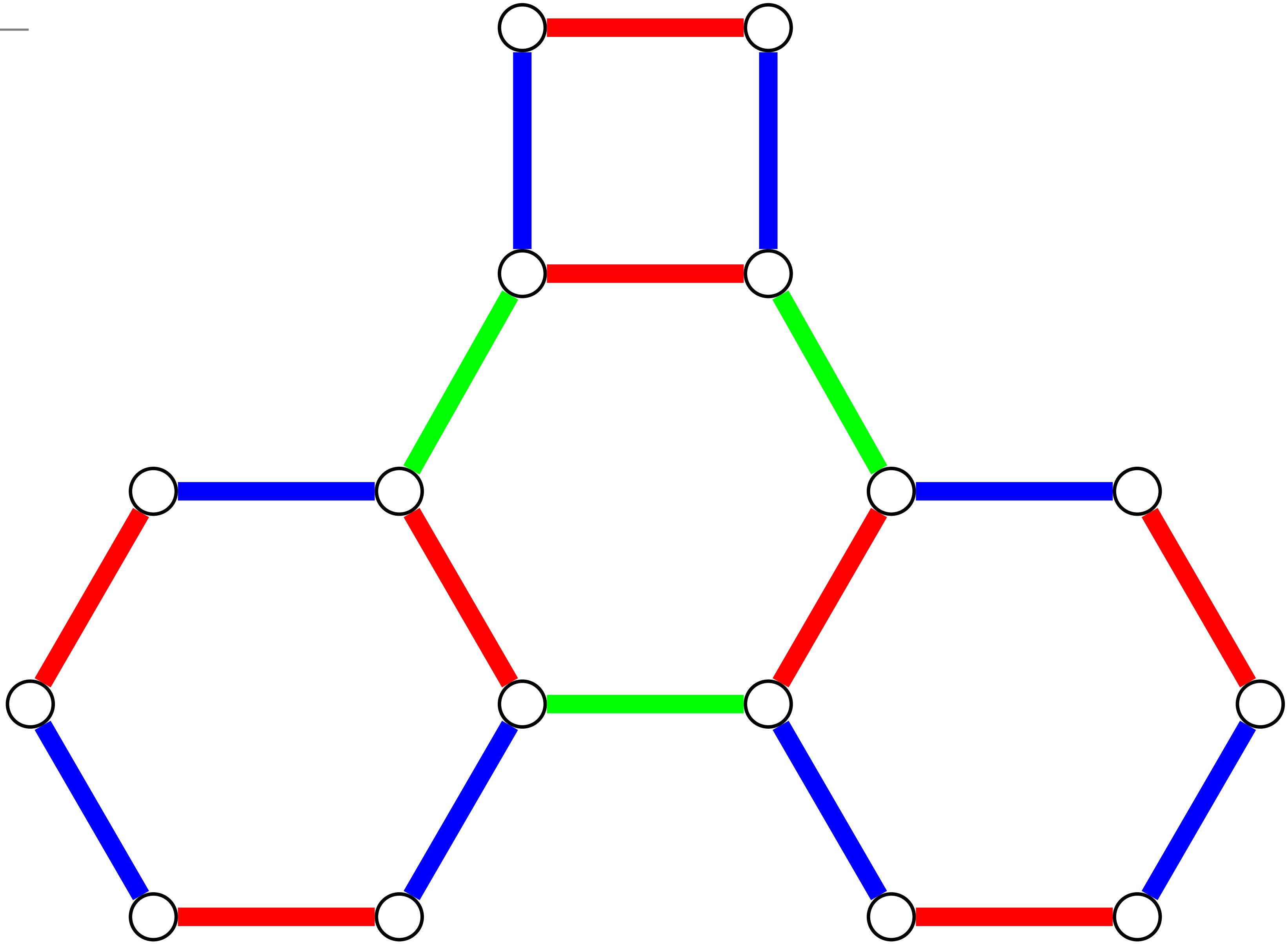

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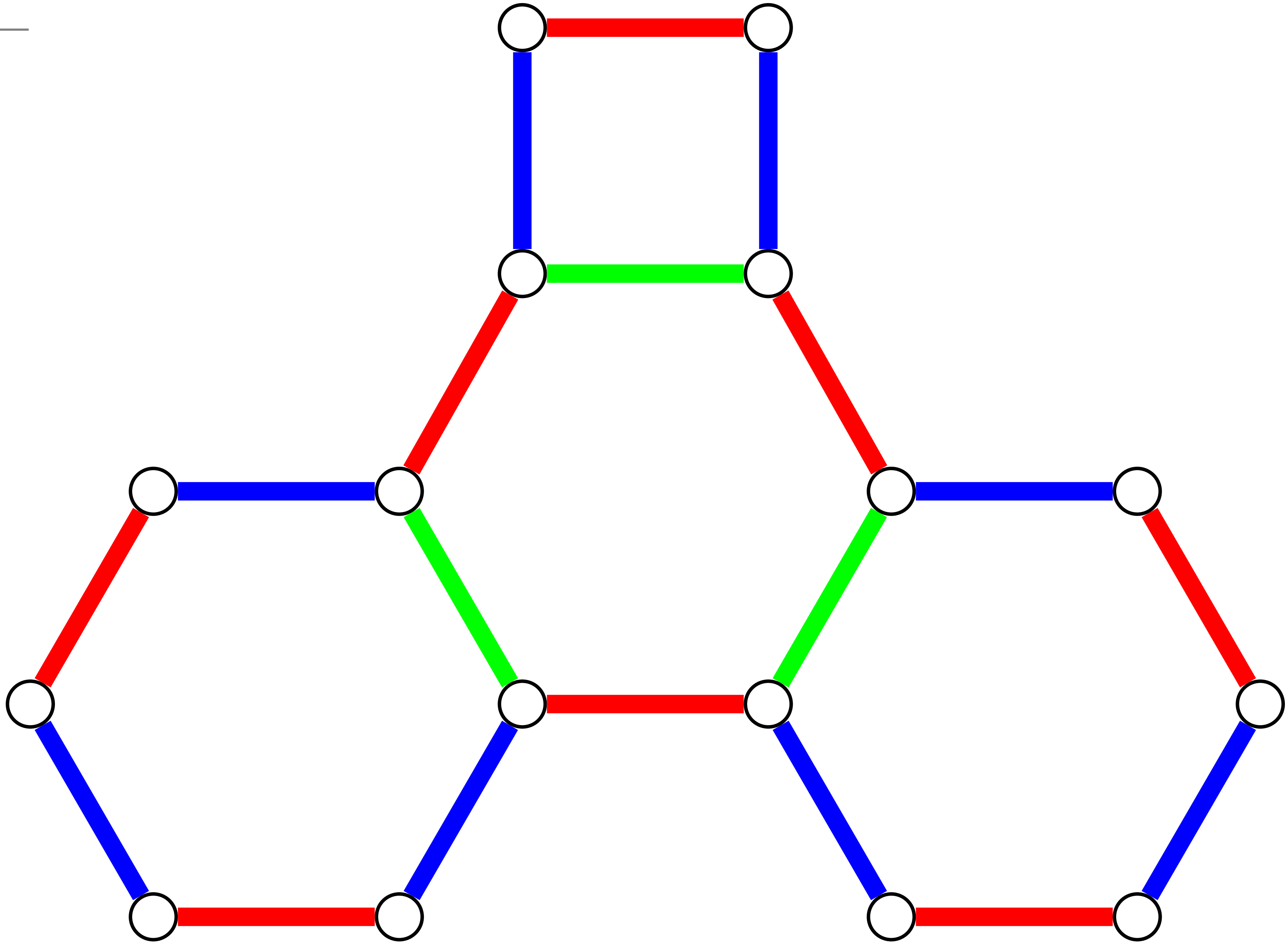
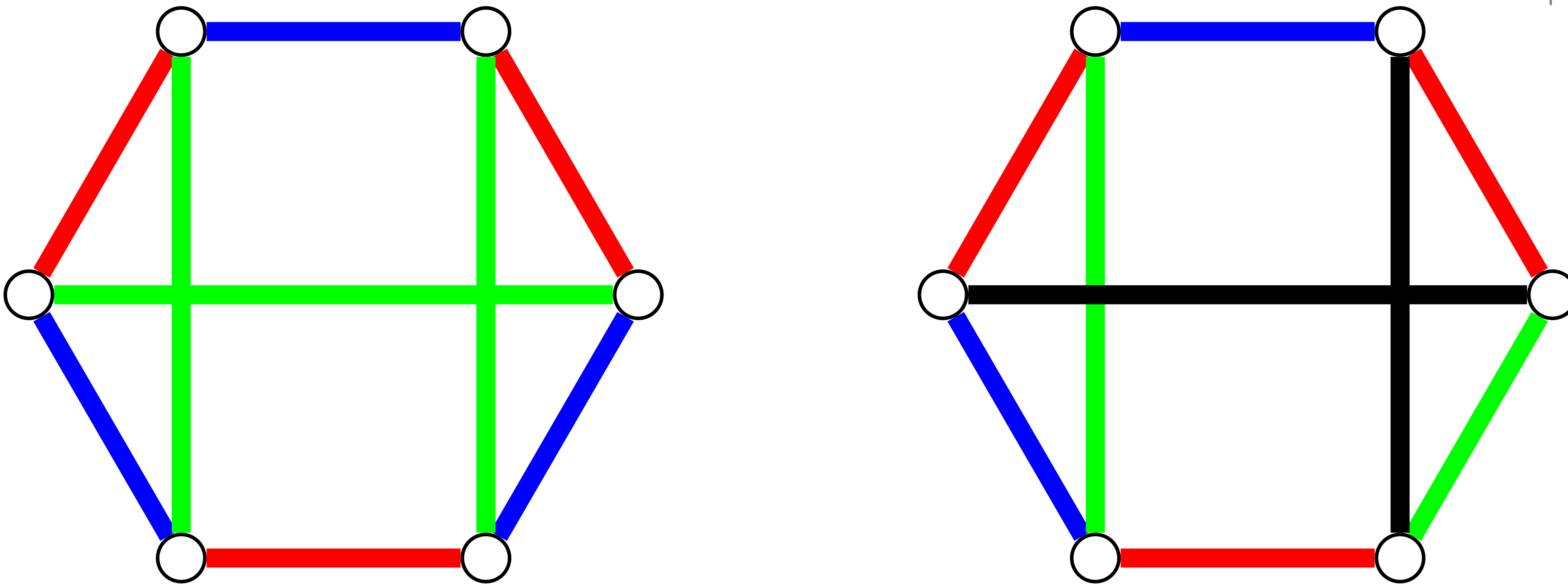
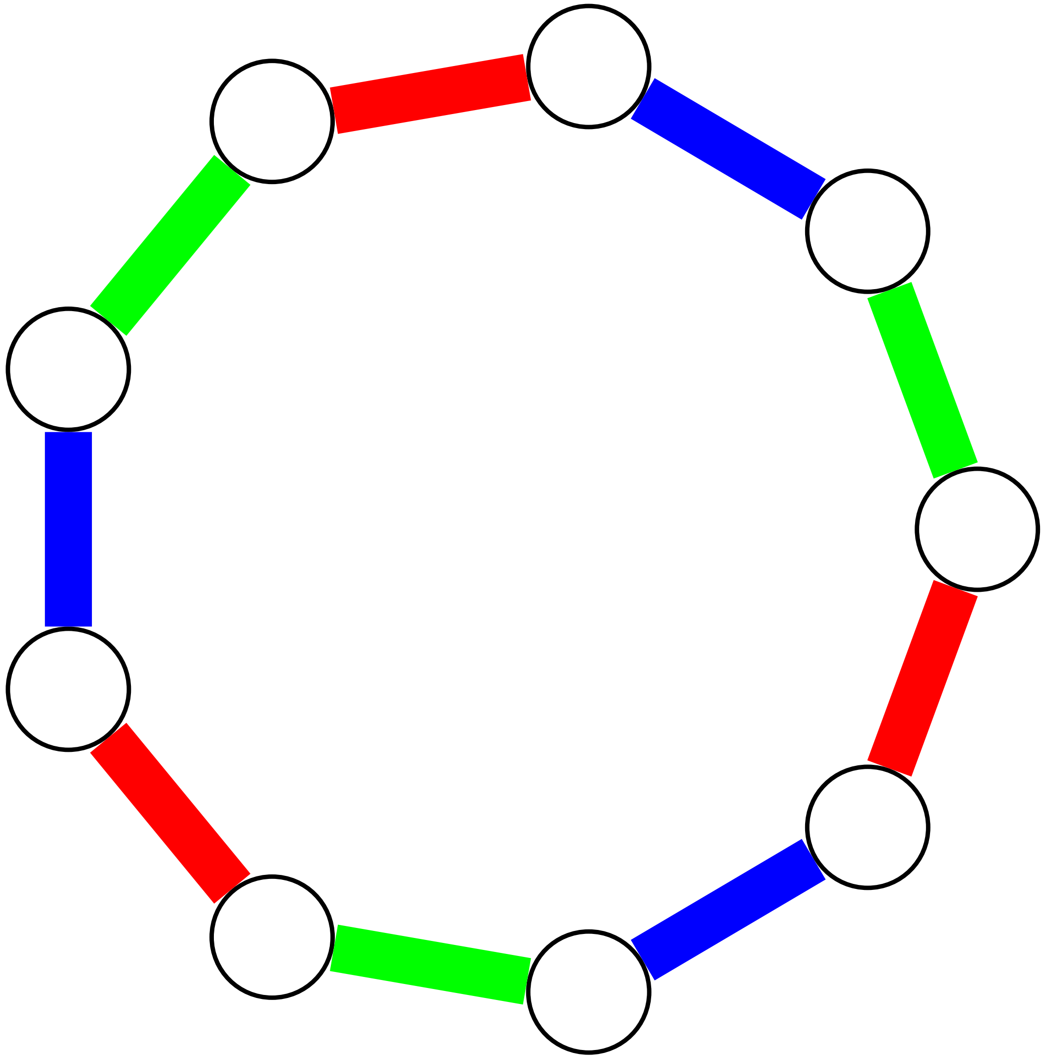


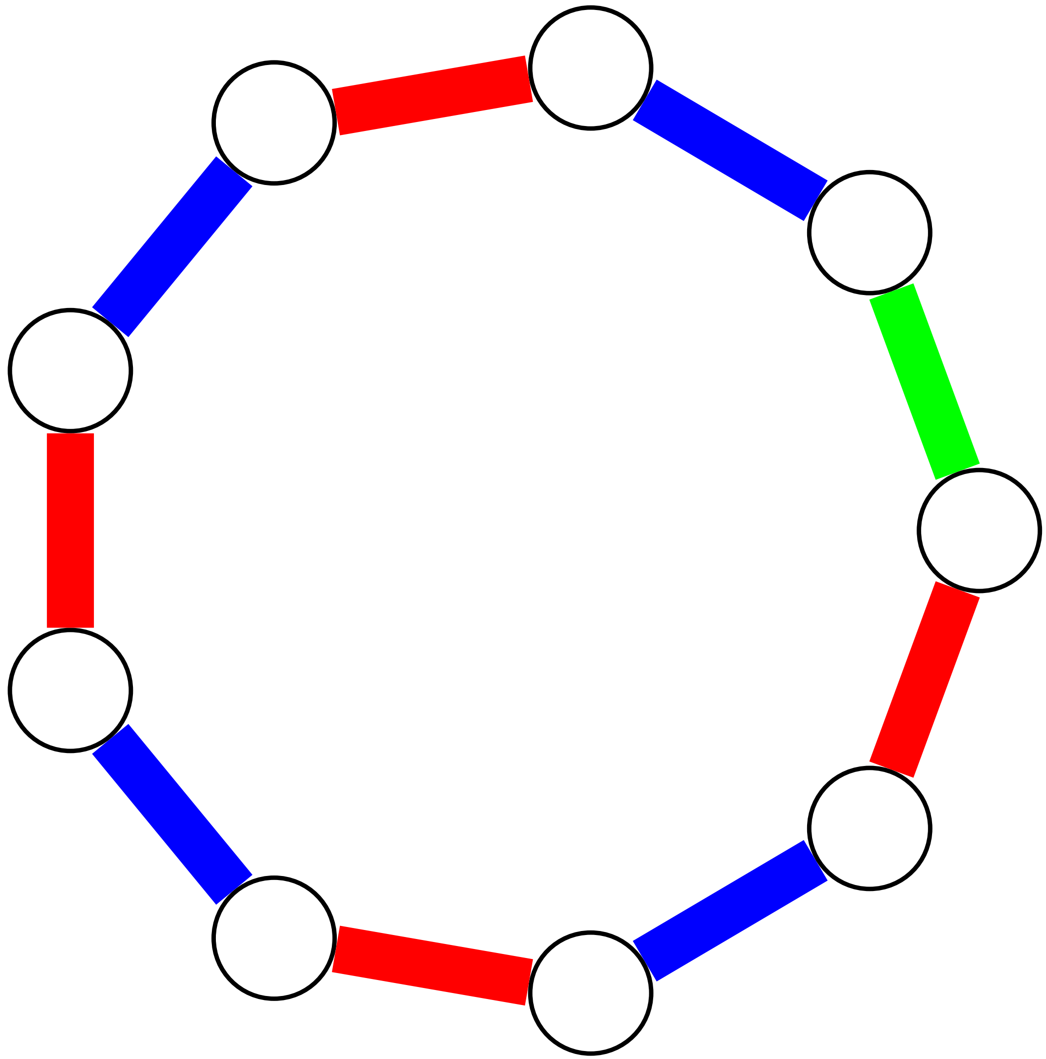
$C_3 \square K_2$



C_9



C_9 - another coloring



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- A (vertex- or edge-) coloring c of a graph G breaks an automorphism φ of G if φ does not preserve colors of c.
- How many colors we need in a coloring that breaks every non-trivial automorphism of G?.

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- Another possibility: endomorphisms instead of automorphisms [W. Imrich, R. Kalinowski, F. Lehner and M. Pilśniak].

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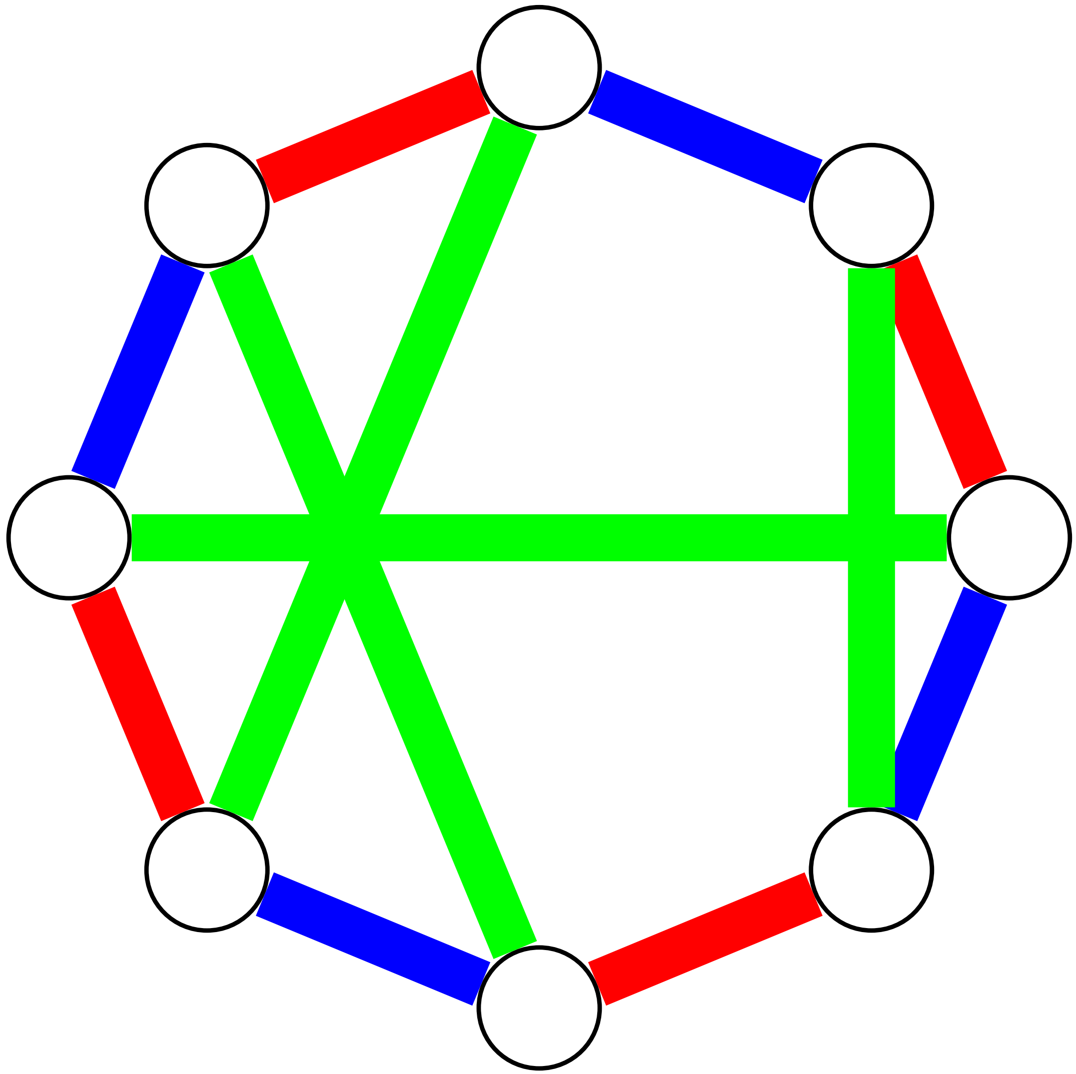
- a contradiction.

$$\chi'_D = 3, \mu = 4$$



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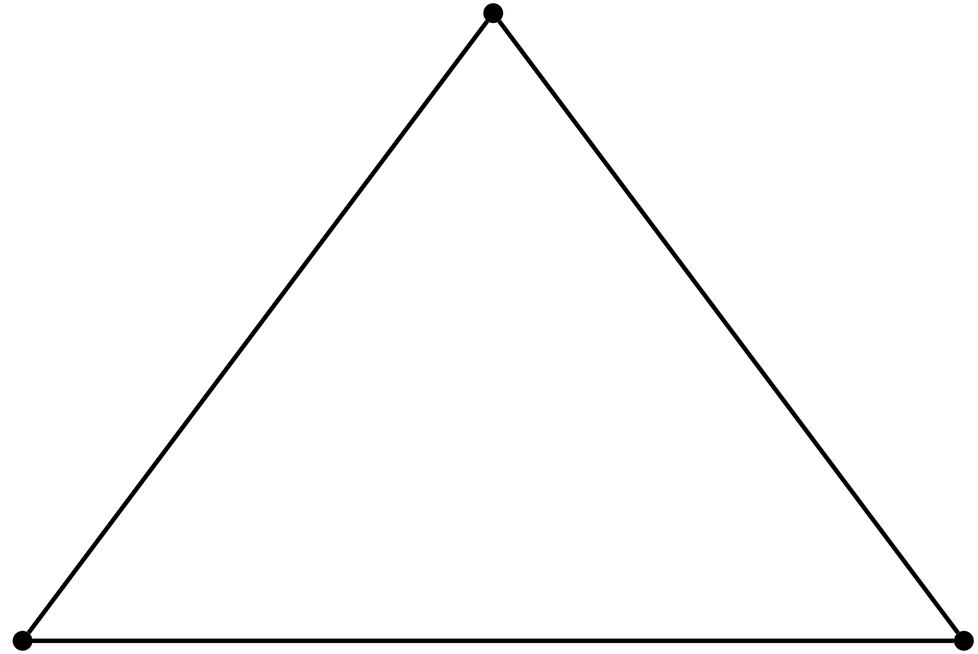
On decomposing regular graphs into locally irregular subgraphs, European Journal of Combinatorics, 49 (2015), 90–104.

Motivation



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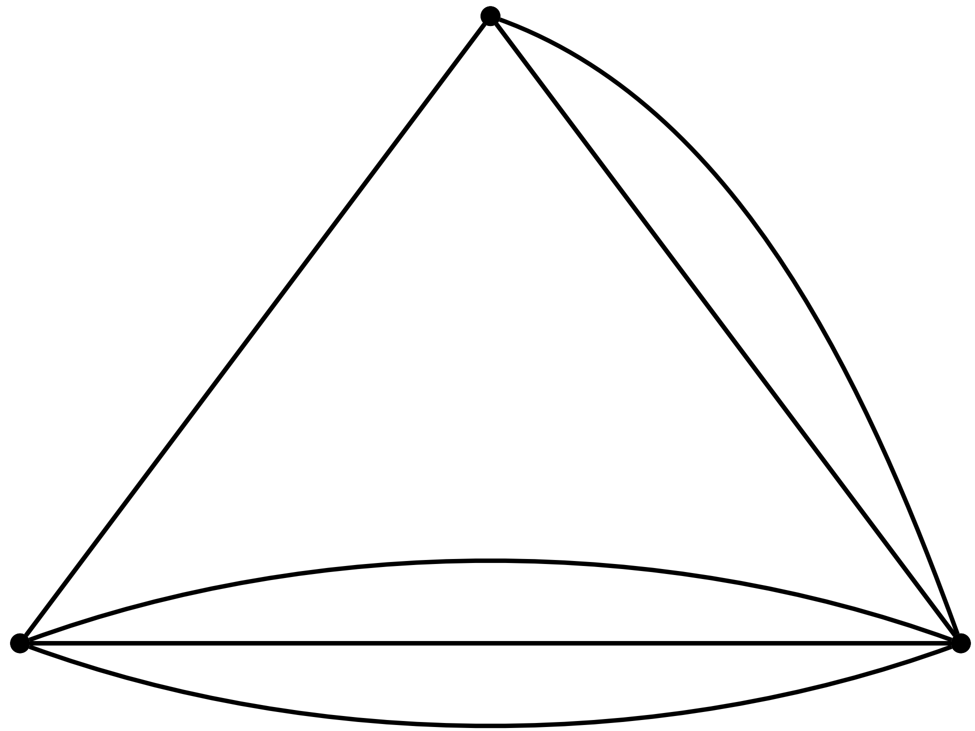

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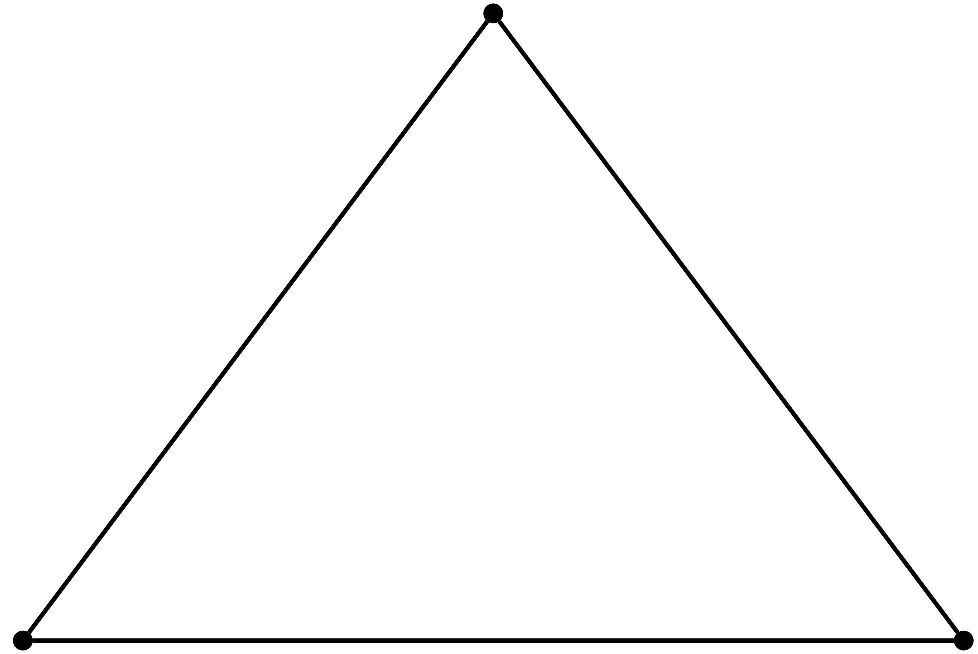
- Even for trees is not completely solved.

Irregularity strengh and coloring



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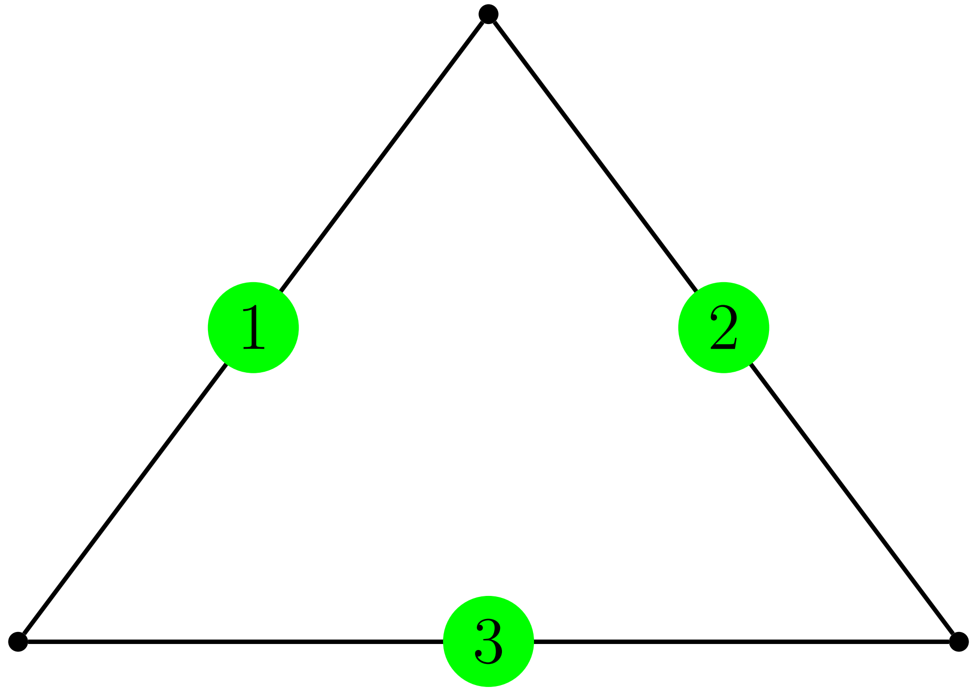

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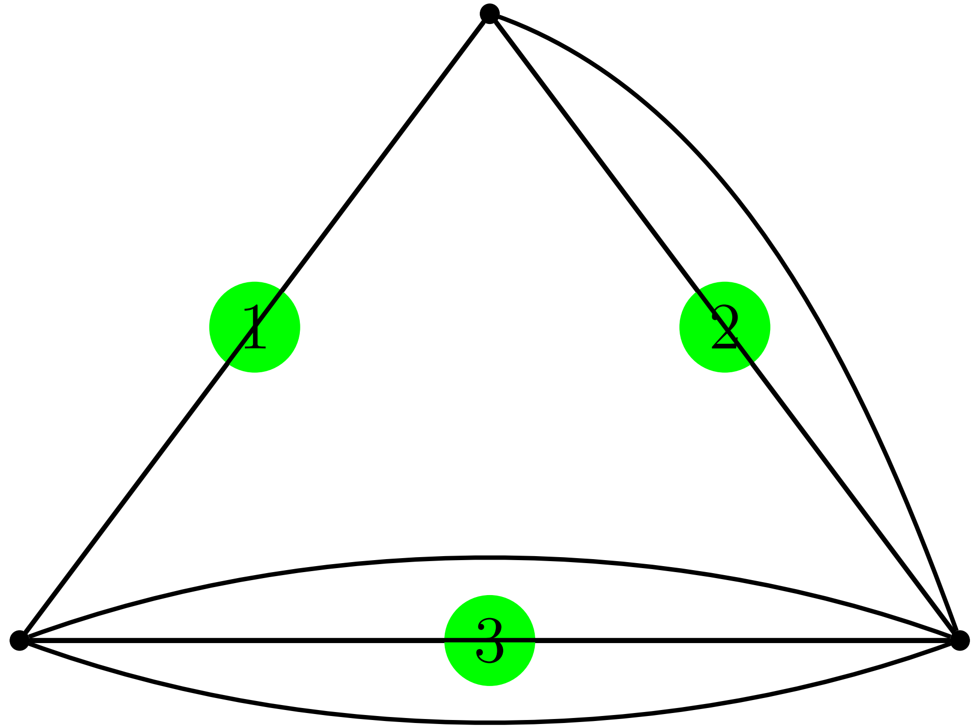

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  $\sigma(x) \neq \sigma(x)$.
- irregularity strength is minimum k such that there exists an f distinguishing all vertices.

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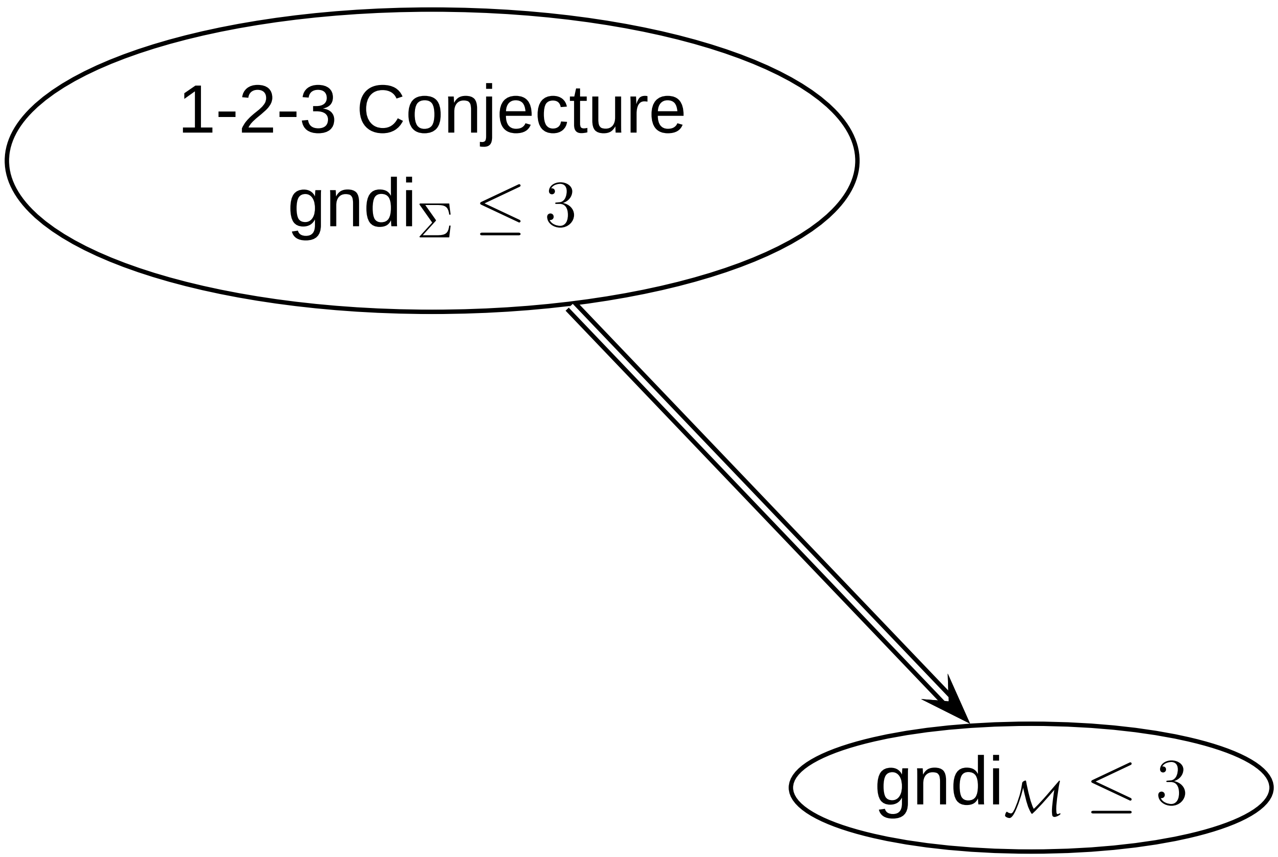

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- Such a decomposition (into k graphs) may be considered as a coloring with k colors such that every color class induces a locally irregular subgraph in G.

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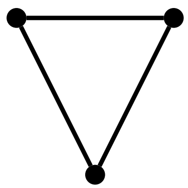
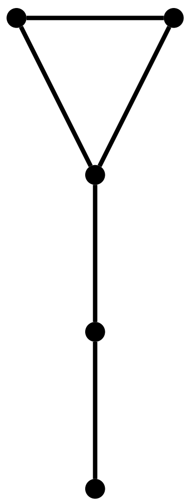
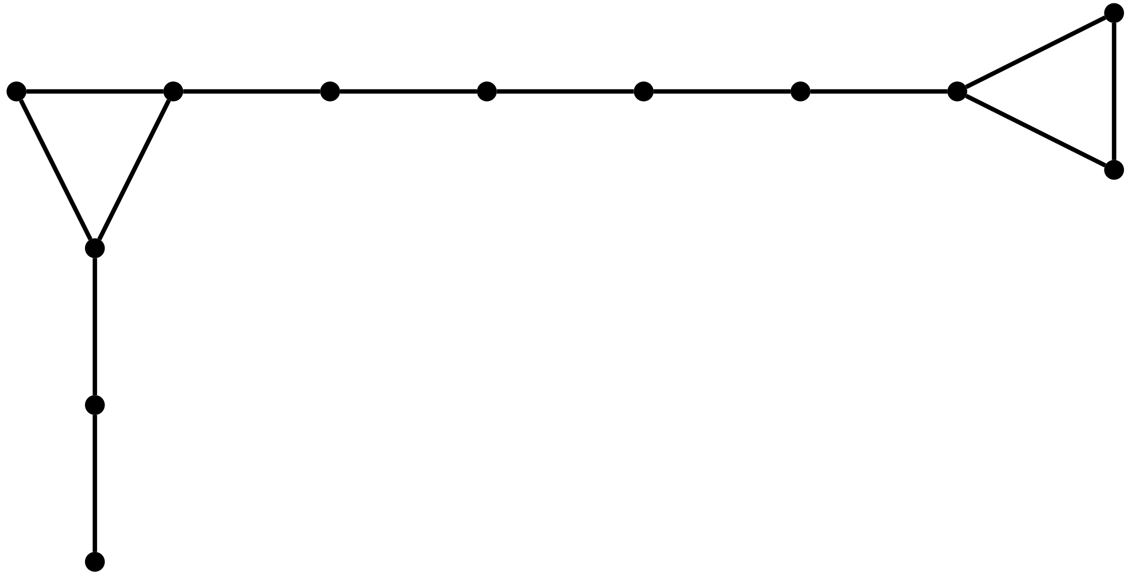


Family $\mathcal{F}$; adding even paths



... adding odd paths with a triangle ...



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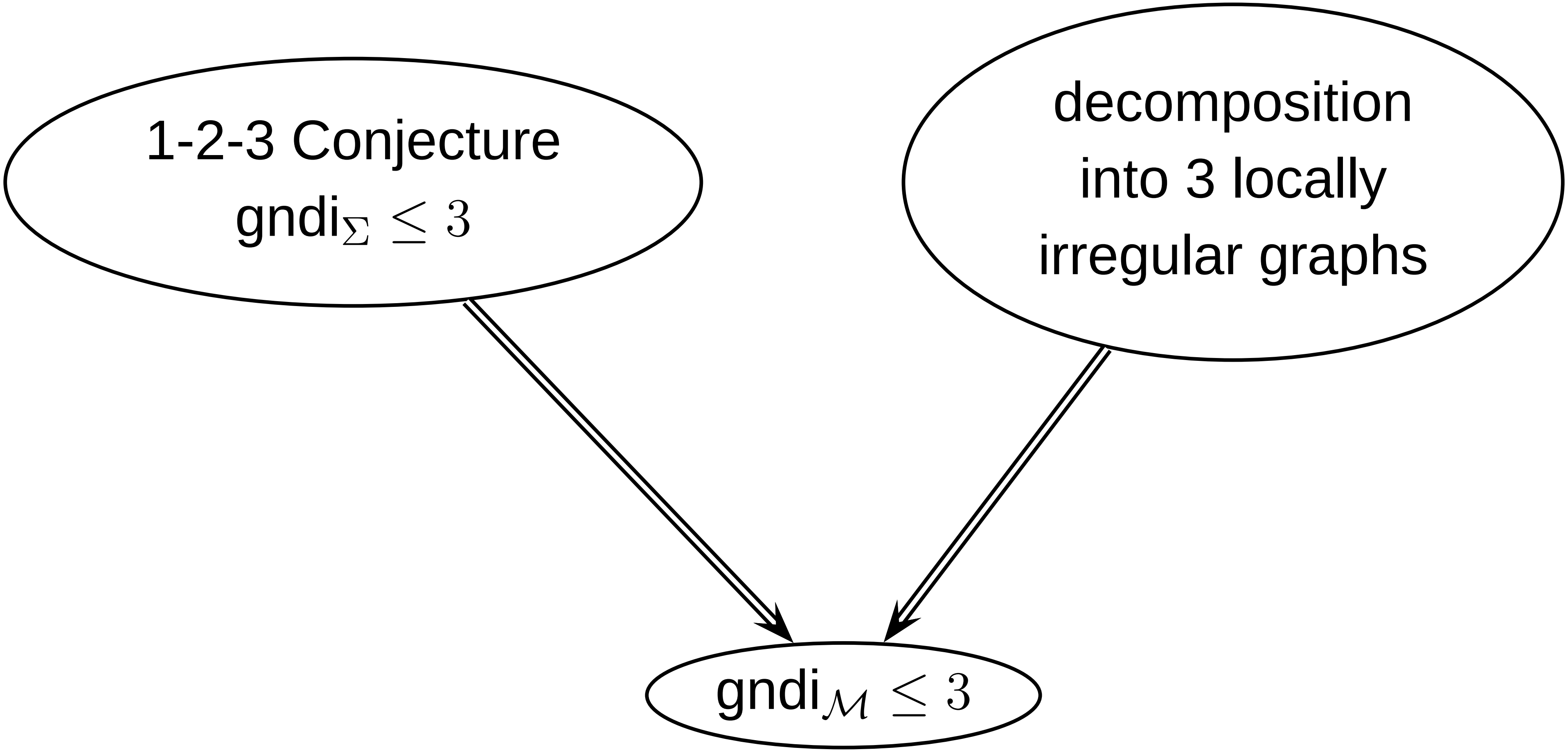
... 3 conjectures ...



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- linear algorithm for determine how many subgraphs we need in the case of trees [O. Baudon, J. Bensmail and E. Sopena, 2013]

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