About convergence to equilibrium for non-reversible Markov processes

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Consider a Markov generator L admitting a reversible probability μ. Let $(P_t)_{t\geqslant 0}$ be the associated semi-group and denote by $\|\!| \cdot |\!\|$ the operator norm in $\mathbb{L}^2(\mu)$. For any time $t \geqslant 0$, we have $\|\!| P_t |\!\| = \exp(-\lambda t)$, where $\lambda \geqslant 0$ is the spectral gap of $-L$. The goal of this mini-course is to begin the investigation of what happens when μ is only assumed to be an invariant probability of L.

We begin by justifying the interest of non-reversible processes by showing how second order Markov chains can be used to improve the speed of convergence to equibrium. Next we investigate in details two toy models of hypocoercive diffusions, corresponding to the generators $L_a = y\partial_x + a\partial_y^2 - y\partial_y$ and $y\partial_x - ax\partial_y + \partial_y^2 - y\partial_y$, respectively on $(\mathbb{R}/(2\pi\mathbb{Z})) \times \mathbb{R}$ and $\mathbb{R} \times \mathbb{R}$, where $a > 0$. We will see how to compute explicitely the operator norms of the associated semi-group from some properties of their spectral decompositions. In particular in small times $t > 0$, $\ln(\|\!| P_t |\!\|)$ is of order t^3 and not of order t as in the situation of reversible processes. Surprisingly, binomial and Poisson laws make an appearance in this purely continuous framework. Finally we will discuss about with some perspectives on finding an alternative approach to hypocoercivity, in discrete or continuous settings.