On Kac's chaos and Kac's program in kinetic theory

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In this talk I will present some (partially joint) works by (with) C. Mouhot, B. Wennberg, M. Hauray, K. Carrapatoso which answer several conjectures raised fifty years ago by Kac in his famous work "Foundations of Kinetic theory". There Kac introduced a many-particle stochastic process which, for chaotic data, converges to the spatially homogeneous Boltzmann equation in the limit of infinity number of particles. I will explain how we can answer the three following questions:

(1) prove the propagation of chaos for realistic microscopic interactions (hard spheres and true Maxwell molecules);

(2) relate the time scales of relaxation of the stochastic process and of the limit equation by obtaining rates independent of the number of particles;

(3) prove the convergence of the many-particle entropy towards the Boltzmann entropy of the solution to the limit equation (microscopic justification of the H-theorem of Boltzmann in this context).

These results crucially rely on

(1) some quantitative estimates on several formulations of Kac's chaos and its link with the (stronger) notion of Entropic chaos;

(2) a quantitative Glivenko-Cantelli theorem for the empirical law of large numbers associated to random vectors which coordinates are not i.i.d. random variables but they are distributed according to a sequence of tensor product measures *conditioned to the Boltzmann's spheres*;

(3) a new theory of quantitative uniform in time estimates of propagation of chaos for stochastic process.