Applications of Lie pseudo-groups to geometry of differential equations

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Abstract. Élie Cartan's structure theory of Lie pseudo-groups provides a convenient and powerful tool to study geometry of differential equations (DEs). The theory is based on a possibility to characterize transformations from a Lie pseudo-group in terms of a finite number of invariant differential 1-forms called Maurer-Cartan forms of the pseudo-group. Unlike computational technique of Sophus Lie's infinitesimal method, Cartan's approach does not use analysis and integration of over-determined systems of partial DEs and allows one to compute Maurer-Cartan forms for symmetry pseudo-groups of DEs by means of operations of linear algebra and differentiation. The Maurer-Cartan forms contain full information about their pseudo-group. In particular, they give all differential invariants of the pseudo-group, thus providing a complete and efficient algorithm for solving equivalence and symmetry classification problems for DEs of physical and mathematical significance.

In the talk, we apply Cartan's method to show that the generalized Saxton-Hunter equation is locally equivalent to the Euler-Poisson equation and thus to prove integrability of the generalized Saxton-Hunter equation in quadratures.

Also, we discuss applications of Cartan's structure theory of Lie pseudo-groups to the theory of coverings of DEs. Coverings (or Wahlquist–Estabrook prolongation structures, or zero-curvature representations, or integrable extensions, etc.) are of great importance in geometry of DEs. The theory of coverings is an adequate universum for dealing with nonlocal symmetries and conservation laws, inverse scattering constructions for soliton equations, Bäcklund transformations, recursion operators, and deformations of nonlinear DEs. A standard approach to finding coverings is developed by Wahlquist and Estabrook. It was designed to apply to equations with two independent variables. Extending the method to DEs with three or more independent variables is a difficult problem.

In the talk we use Cartan's method to find contact integrable extensions, coverings with spectral parameters and a recursion operator for the four-dimensional Martínez Alonso - Shabat equation.