Inverse problem on conservation laws: theory and applications

Roman Popovych
Wolfgang Pauli Institute, Vienna, Austria
Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine

Abstract:

The standard (direct) problem on conservation laws is roughly formulated as follows: given a system of differential equations, study its conservation laws. At the same time, several important applications, e.g., the parameterization of differential equations, need solving the inverse problem, which has received less attention so far although. In a narrow sense, the inverse problem on conservation laws intuitively means deriving the general form of systems of differential equations with a prescribed set of conservation laws. More generally, it can be interpreted as the study of properties of systems for which something is known about their conservation laws. We discuss which data of conservation laws are appropriate for precisely posing various inverse problems on conservation laws. Several such problems are completely solved, including the inverse problem on integrating factors of single ordinary differential equations. Using the class of (1+1)-dimensional evolution equations, we demonstrate how the solution of the inverse problem on conservation laws can help to solve the direct problem for the same class of equations. Results on a specific case of inverse problems that is associated with families of conservation laws parameterized by arbitrary functions are applied to conservative parameterization of the vorticity equation. The generalized second Noether theorem is shown to well fit within the framework presented.