Wandering Seminar
Ergodic Theory & Dynamical Systems
AGH University of Science and Technology
Kraków, 20–23 November 2014

`wms.mat.agh.edu.pl/~wsetds3/wandering.html`

Abstracts of talks

November 17, 2014

Mini-course, Friday–Sunday, November 21–23

Inducing Techniques in Thermodynamic Formalism

Henk Bruin and Mike Todd

The theory of thermodynamic formalism was introduced into dynamics by Sinai, Ruelle and Bowen, initially for hyperbolic systems, to give the ideas of thermodynamic pressure, equilibrium states and phase transitions on a mathematical basis. Non-(uniformly) hyperbolic system can frequently be accelerated so as to produce a hyperbolic system (with usually countably many branches), and this technique is called inducing. It requires particular adaptations which will be discussed in this course, especially for maps of the interval and symbolic (sub)shift systems.
Topics to be covered include:

- basic notions of thermodynamic formalism and phase transitions;
- shift spaces, substitution shifts;
- relevant notions of interval dynamics;
- aspects of hyperbolic expanding maps with countably many branches;
- applications to multifractal analysis;
- physics motivation with quasi-crystals.

Introductory lecture, Thursday, November 20

Dominik Kwietniak

This talk is intended to introduce/refresh some prerequisites for the minicourse presented by Henk and Mike. We will cover the following topics:

- Symbolic dynamics (shifts of finite type).
- Substitutions: Thue-Morse and Fibonacci. Sturmian subshifts.
- Topological & metric entropy and the Variational Principle.

Talks (Friday–Sunday, November 21–23 (in chronological order))

Gauss, Rauzy-Veech and resonances

Mark Pollicott

The Rauzy-Veech map is a higher dimensional analogue of the classical Farey map of the interval. The associated accelerated map (by suitably iterating the original map) is analogous to the Gauss map (or continued fraction map) of the interval. Some aspects of the dynamical behaviour of such maps can often be understood via the properties of a Ruelle-Perron-Frobenius type operator. Of particular interest is undertanding suitable suspension flows, which inform the behaviour of the geodesic flow on the modular surface and the Teichmuller geodesic flow via characterisitc values called resonances.

Entropy, rotation sets, inverse limits and attractors

Jan Boroński

I shall discuss some new results (joint work with P. Oprocha) conerning the dynamics of graph maps that give hereditarily indecomposable inverse limits. My particular interest will be in circle maps that "generate" R.H. Bing's pseudo-circle, and motivations from the study of strange attractors in surface dynamics.

Loops of transitive interval maps

Michał Misiurewicz

In the paper [S. Kolyada, M. Misiurewicz and L. Snoha, Spaces of transitive interval maps, Ergod. Th. & Dynam. Sys., posted electronically on August 5, 2014] we investigated topology of spaces of continuous topologically transitive interval maps. Let T_n denote the space of all transitive maps of modality n. For every $n > 0$, in the union of T_n and T_{n+1} we constructed a loop (call it L_n), which is not contractible in in this space. One of the main open problems left in that paper was whether L_n can be contracted in the union of more of spaces T_i.
Since all elements of the loops L_n are maps of constant slope, we are studying the spaces TCS_n of transitive maps of modality n and constant slope. We show that for every $n > 1$ the loops L_n and L_{n+1} can be contracted in the union of TCS_n, TCS_{n+1} and TCS_{n+2}. Moreover, the loop L_1 can be contracted in the union of TCS_1, TCS_2 and TCS_4.
Additionally, we describe completely the topology (and geometry, in a certain parametrization) of the spaces TCS_1, TCS_2 and their union. In particular, we show that this union is homotopically equivalent to the circle.

Countably Piecewise Monotone Maps and Constant Slope

Samuel Roth

I will discuss maps of the interval which are piecewise-monotone and piecewise-continuous with countably many pieces. I present a necessary and sufficient condition for the existence of a nondecreasing semiconjugacy to a map of constant slope, and then give examples which violate this criterion. Finally, in answer to a question by Bobok and Bruin, I present some preliminary results for constant slope maps on the real line.

Subdiagrams and invariant measures on Bratteli diagrams

Olena Karpel

We study ergodic finite and infinite measures defined on the path space X_B of a Bratteli diagram B which are invariant with respect to the tail equivalence relation on X_B. Our interest is also focused on measures supported by vertex and edge subdiagrams of B. We give several criteria when a finite invariant measure defined on the path space of a subdiagram of B extends to a finite invariant measure on B. Given a finite ergodic measure on a Bratteli diagram B and a subdiagram B' of B, we find the necessary and sufficient conditions under which the measure of the path space $X_{B'}$ of B' is positive. For a class of Bratteli diagrams of finite rank, we determine when they have maximal possible number of ergodic invariant measures. The case of diagrams of rank two is completely studied. We include also an example that explicitly illustrates the proved results. The talk is based on joint work with M. Adamska, S. Bezuglyi and J. Kwiatkowski.

(C, F)-construction, joinings and prime factors of simple actions

Anton Solomko

I will explain the (C,F)-construction of measure preserving actions, which is an algebraic counterpart of classical cutting-and-stacking technique, and demonstrate some applications of this technique to the theory of joinings of dynamical systems. In particular, (C,F)-formalism combined with Ornstein's technique of "random spacers" allows to produce probability preserving simple actions with centralizers prescribed in advance. I will present a recent joint work with A. Danilenko, where we construct simple mixing actions with uncountably many prime factors.

Stability of the measure of maximal entropy for piecewise monotonic interval maps

Peter Raith

Consider a piecewise monotonic map $T : [0, 1] \to [0, 1]$. This means that there exists a finite partition $\mathcal{Z}$ of $[0, 1]$ into pairwise disjoint open intervals with $\bigcup_{Z\in\mathcal{Z}} \overline{Z} = [0, 1]$ such that $T|_Z$ is continuous and strictly monotonic for all $Z \in \mathcal{Z}$. Observe that T need not be continuous at the endpoints of intervals of monotonicity. A T-invariant Borel probability measure μ is said to be a measure of maximal entropy if its measure-theoretic entropy coincides with the topological entropy of T. Assume that T has a unique measure μ of maximal entropy.
One calls two piecewise monotonic maps "close", if they have the same number of intervals of monotonicity and their graphs are "close" considered as subsets of $\mathbb{R}^2$. In general the measure of maximal entropy need not be continuous. A certain number is associated to T, and if the topological entropy of T is larger than this number every piecewise monotonic map $\widetilde{T}$ sufficiently close to T has a unique measure $\mu_{\widetilde{T}}$ of maximal entropy. Furthermore, if $\widetilde{T}$ tends to T, then $\mu_{\widetilde{T}}$ converges to μ in the w*-toplogy.
For unimodal maps (continuous piecewise maps with two intervals of monotonicity) T with positive entropy the measure of maximal entropy is always continuous. In this case T always has a unique measure of maximal entropy. Also the case of monotonic mod one transformations T is investigated. If T has positive entropy and a unique measure of maximal entropy, then the measure of maximal entropy is continuous. A slightly different topology on monotonic mod one transformations is also considered.

Entropy as infimum over finite subsets

Bartosz Frej

We study fairly simple, but general approach to the notion of entropy. The formula works well in classic systems, not only for a single measure preserving map, but even for arbitrary amenable group actions, and it is also applicable to more general group actions. Both measure-theoretic and topologic cases are considered.

Projections of percolations

Michał Rams

I'll be speaking about results (joint with Karoly Simon) on the projections of percolations. Fractal percolation is the simplest random fractal construction. We prove that for such sets almost surely we have a result significantly strengthening the classical Marstrand theorem. That is, if the dimension of the percolation fractal is greater than 1 then almost surely all the linear projections of this set contain an interval, while if the dimension is not greater than 1 then almost surely all the linear projections have the same dimensin as the original set. I will also present several other results of this type.

Finitely Computable Functions in Sofic Möbius Number Systems

Tomáš Vávra

We study functions that are computable by a finite-state transducers in Möbius number systems with sofic expansion subshifts. We show that if a function F is analytic on an extended real line, and computable by a finite-state transducer, then F is a Möbius transformation, e.g. $F(x) = \frac{ax+b}{cx+d}$. Furthemore we show that unlike in modular Mobius systems, in bimodular systems, not every rational Möbius transformation is computable by a finite state transducer.

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